Math 483 HW 9 2023. This is a HARD HW! Due Thursday, Sept. 28. Quiz 4 is Friday, Oct. 2. E(Y), V(Y), mgf m(t) for discrete and continuous RVs, find c so f(y) = c g(y) integrates to one, find probabilities given f(y) or F(y). Find f(y) from F(y) and vice verca. Normal table. **Two Pages problems A)-G).**

A) 4.111a Suppose that Y has a gamma distribution with parameters α and β. If a is any positive or negative value such that $\alpha + a > 0$, show that

$$E(Y^a) = \frac{\beta^a \Gamma(\alpha + a)}{\Gamma(\alpha)}.$$

comment: Use the kernel technique. Find the gamma pdf in the back of the book.

B) 4.133ab Consider a random variable Y with pdf

$$f(y) = \begin{cases} cy^2(1-y)^4, & \text{if } 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of c that makes $f(y)$ a pdf.
b) Find $E(Y)$.
comment: Match the pdf with the pdf of the beta distribution given in the back of the book. See p. 194. To find E(Y), use p.195.

C) 4.136 Suppose that Y is an exponential random variable with pdf

$$f(y) = \begin{cases} \left(\frac{1}{\theta}\right) e^{-y/\theta} & \text{if } y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the moment generating function for Y (either by integrating or using the mgf of a Gamma distribution).
b) Use the answer from a) to find $E(Y)$ and $V(Y)$.
comment: See p. 202 and ex. 4.13 on p. 203. To find E(Y) and E(Y^2), use p. 139 (or the middle of p. 203).

D) 4.145bc Suppose that Y has pdf

$$f(y) = \begin{cases} e^y, & \text{if } y < 0 \\ 0, & \text{otherwise.} \end{cases}$$

b) Find the moment generating function of Y using the fact that e^{ay}/a is the antiderivative of e^{ay} if $a > 0$.
c) Using the mgf, find $V(Y)$.

E) 5.4b Given below is the joint probability function associated with data obtained in a study of car accidents in which a child under age 5 was in the car and at least one fatality occurred. Define

$$Y_1 = \begin{cases} 0, & \text{if child survived} \\ 1, & \text{otherwise} \end{cases} \quad \text{and} \quad Y_2 = \begin{cases} 0, & \text{if no belt used} \\ 1, & \text{if adult belt used} \\ 2, & \text{if car}-\text{seat belt used.} \end{cases}$$

Then Y_1 is the number of fatalities per child and since children's car seats use two belts, Y_2 is the number of seatbelts in use at the time of the accident.

$p(y_1, y_2)$		$y_1 = 0$	$y_1 = 1$	
y_2	0	0.38	0.17	0.55
	1	0.14	0.02	0.16
	2	0.24	0.05	0.29
		0.76	0.24	1.00

b) Find $F(1, 2)$.
comment: See p. 226-227. Ignore the interpretation.

F) 5.9a Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} k(1 - y_2), & \text{if } 0 \le y_1 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of k.
comment: Use iterated integrals to evaluate the double integral.

G) 5.10a Let Y_1 and Y_2 have joint pdf

$$f(y_1, y_2) = \begin{cases} k, & \text{if } 0 \le y_1 \le 2,\ 0 \le y_2 \le 1,\ 2y_2 \le y_1 \\ 0, & \text{otherwise.} \end{cases}$$

That is, Y_1 and Y_2 are uniformly distributed over the region inside the triangle bounded by $y_1 = 2$, $y_2 = 0$ and $2y_2 = y_1$.
a) Find k.
comment: Use iterated integrals to evaluate the double integral or use geometry.