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Math 483 exam 2, Fall 2006 Name____________________

y	0	1	2	3
p(y)	0.08	0.42	0.32	0.18

1) Let the discrete random variable Y have a probability function given by the table above.

a) Find E(Y). $= \sum y\, p(y) = 0(.08) + 1(.42) + 2(.32) + 3(.18)$

$= \boxed{1.6}$

b) Find E(Y^2). $= \sum y^2 p(y) = 0^2(.08) + 1^2(.42) + 2^2(.32) + 3^2(.18) = \boxed{3.32}$

c) Find the standard deviation of Y. $= \sqrt{EY^2 - (EY)^2} = \sqrt{3.32 - (1.6)^2}$

$= \sqrt{0.76} = \boxed{.8718}$

d) Find the moment generating function of Y.

$$\boxed{m(t) = \sum e^{ty} p(y) = e^{0t}(.08) + e^{1t}(.42) + e^{2t}(.32) + e^{3t}(.18)}$$

$= .08 + .42e^{t} + .32e^{2t} + .18e^{3t}$

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Math 483 exam 2, 2010

2) SAT scores X follow a normal distribution with mean $\mu = 1000$ and standard deviation $\sigma = 150$ ~~days~~.

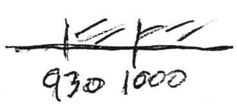
a) How high must a student score in order to place in the top 5% of all students taking the SAT?

$X = \mu + \sigma Z = 100 + 150(1.645) = \boxed{1246.75}$

	.04	.05
1.6	.0505	.0495

$\frac{1.64 + 1.65}{2} = 1.645$

b) Find the probability that X will be greater than 930.



$Z = \frac{930 - 1000}{150} = -0.47$

$= 1 -$

$= .46 - 1$

	.07
.4	.3192

$P(X > 930) = 1 - P(Z > .47) = 1 - .3192 = \boxed{.6808}$

c) Find the probability that X will be between 800 and 1300.

$\frac{800 - 1000}{150} = -1.33$ $\frac{1300 - 1000}{150} = 2.00$

	00	03
1.3		.0918
2.0	.0228	

prob $= 1 - P(Z > 1.33) - P(Z > 2) = 1 - .0918 - .0228 = \boxed{0.8854}$

3) Suppose that the moment generating function (mgf) of a random variable Y is

$$m(t) = \frac{1}{1 - \lambda t}$$

where $\lambda > 0$ is a **known constant.** Using the mgf $m(t)$, find $m'(t)$ and $E(Y)$.

$m'(t) = \frac{d}{dt}(1 - \lambda t)^{-1} = -1(1 - \lambda t)^{-2}(-\lambda) = \lambda(1 - \lambda t)^{-2}$

$E(Y) = m'(0) = \lambda(1)^{-2} = \boxed{\lambda}$

quotient rule

$\left(\left(\frac{n}{d}\right)' = \frac{dn' - d'n}{d^2} = \frac{0 - (-\lambda)}{(1 - \lambda t)^2} \right)$

Math 483 Exam 2, Fall 2003

2) The lengths of human pregnancy X follow a normal distribution with mean $\mu = 266$ and standard deviation $\sigma = 16$ days.

a) What is the length of pregnancy such that 90% of pregnancies are longer?

.1 .9

-1.28

$$\begin{array}{c|c} & .08 \\ \hline 1.2 & .1003 \end{array} \quad z_0 = -1.28$$

$$x_0 = \mu + \sigma z_0 = 266 + 16(-1.28) = \boxed{245.52}$$

b) Find the probability that X will be greater than 273.

266 273

$$z = \frac{273-266}{16} = .44$$

$$\begin{array}{c|c} & .04 \\ \hline .4 & .3300 \end{array}$$

$\boxed{.3300}$

c) Find the probability that X will be between 250 and 282.

250 282

$$\frac{250-266}{16} = -1.00, \quad \frac{282-266}{16} = 1.00$$

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$$\begin{array}{c|c} & .00 \\ \hline 1.0 & .1587 \end{array}$$

-1 1

$$\text{prob} = 1 - .1587 - .1587 = \boxed{.6826}$$

3) Suppose that the moment generating function (mgf) of a random variable Y is

$$m(t) = \exp[\lambda(e^t - 1)]$$

where $\lambda > 0$ is a **known constant.** Using the mgf $m(t)$, find $m'(t)$ and $E(Y)$.

$$m'(t) = \exp[\lambda(e^t - 1)]\, \lambda e^t$$

$$m'(0) = \exp(0)\, \lambda e^0 = \boxed{\lambda = EY}$$

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4) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} c\,y, & \text{if } 0 \le y \le 5 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c. $1 = c\int_0^5 y\,dy = c\,\frac{y^2}{2}\Big|_0^5 = \frac{25}{2}c$, $\boxed{c = \frac{2}{25} = .08}$

b) Find $E(Y)$. $= \int_0^5 y\,\frac{2}{25}\,y\,dy = \int_0^5 \frac{2}{25}y^2\,dy = \frac{2}{25}\,\frac{y^3}{3}\Big|_0^5$

$= \frac{2(125)}{25(3)} = \boxed{\frac{10}{3} = 3.3333}$

c) Find $V(Y)$. $EY^2 = \int_0^5 y^2\,\frac{2}{25}\,y\,dy = \int_0^5 \frac{2}{25}y^3\,dy = \frac{2}{25}\,\frac{y^4}{4}\Big|_0^5$

$= \frac{2}{25}\,\frac{25(25)}{4} = \frac{25}{2} = 12.5$

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$V(Y) = EY^2 - (EY)^2 = \frac{25}{2} - \left(\frac{10}{3}\right)^2 = \frac{9(25) - 200}{18} = \boxed{\frac{25}{18} = 1.388}$

e 5) If Y is Poisson with $\lambda = 8$

a) Find $E(Y)$.

$= \boxed{8}$

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b) Find $P(Y = 5)$. $= \frac{\lambda^y e^{-\lambda}}{y!} = \frac{8^5 e^{-8}}{5!} = \boxed{.09160}$

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$1 = c \int_0^2 y_1 dy_1 \int_0^1 y_2 dy_2 = c \frac{y_1^2}{2}\Big|_0^2 \frac{y_2^2}{2}\Big|_0^1$

$= c \frac{4}{2} \frac{1}{2} = c$

6) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} c\, y_1 y_2, & \text{if } 0 \le y_1 \le 2, 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c. $c \int_0^2 \int_0^1 y_1 y_2 \, dy_2 \, dy_1 = c \int_0^2 y_1 \left(\frac{y_2^2}{2}\Big|_0^1 \right) dy_1 = c \int_0^2 \frac{y_1}{2} dy_1$

$= \frac{c}{2} \frac{y_1^2}{2}\Big|_0^2 = c \frac{4}{4} = c = 1$ so $\boxed{c = 1}$

or $c \int_0^1 \int_0^2 y_1 y_2 \, dy_1 \, dy_2 = c \int_0^1 \left(\frac{y_1^2}{2}\Big|_0^2 \right) y_2 \, dy_2 = c \int_0^1 2 y_2 \, dy_2 = 2c \frac{y_2^2}{2}\Big|_0^1$

$= 2\frac{c}{2} = c = 1$ or $c = 1$

b) Find the marginal pdf of Y_1. Include the support.

$f_{Y_1}(y_1) = \int_0^1 f(y_1, y_2) \, dy_2 = \int_0^1 y_1 y_2 \, dy_2 = y_1 \frac{y_2^2}{2}\Big|_0^1 = \frac{y_1}{2}, 0 < y_1 <$

$0 < y_1 < 2$

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c) Find the marginal pdf of Y_2. Include the support.

$f_{Y_2}(y_2) = \int_0^2 f(y_1, y_2) \, dy_1 = \int_0^2 y_1 y_2 \, dy_1 = \left(\frac{y_1^2}{2}\Big|_0^2 \right) y_2 = 2 y_2, 0 < y_2 <$

$0 < y_2 < 1$

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d) Find the conditional pdf of Y_1 given $Y_2 = y_2$, that is, find $f_{Y_1|Y_2=y_2}(y_1|y_2) \equiv f(y_1|y_2)$. Make sure you include the support of the conditional pdf.

$$f_{Y_1|Y_2}(y_1|y_2) = \frac{f(y_1, y_2)}{f_{Y_2}(y_2)} = \frac{y_1 y_2}{2 y_2} = \boxed{\frac{y_1}{2}, 0 < y_1 < 2}$$

(as expected from ind)

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7) Suppose that the joint probability function $p(y_1, y_2)$ of Y_1 and Y_2 is and is tabled as shown.

$p(y_1, y_2)$		y_1 = 1	2	3	4
y_2	1	0.062	0.192	0.176	0.210
	2	0.006	0.011	0.008	0.006
	3	0.083	0.115	0.070	0.061
		.151	.318	.254	.277

← compute

a) Find the marginal probability function $p_{Y_1}(y_1)$ for Y_1.

y_1	1	2	3	4
$P_{Y_1}(y_1)$	.151	.318	.254	.277

b) Find the conditional probability function $p(y_2|y_1)$ of Y_2 given $Y_1 = 2$.

$\left(= \frac{P(2, y_2)}{P_1(2)} = \frac{P(2, y_2)}{.318} \right)$ $\frac{.192}{.318}$ $\frac{.011}{.318}$ $\frac{.115}{.318}$ or -3

y_2	1	2	3
$P(y_2\|2)$	.6038	.0346	.3616

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8) Suppose that Y is a random variable with distribution function

$$F(y) = 1 - e^{-y}$$

for $y > 0$ and that $F(y) = 0$ otherwise. Find $P(Y > 6)$.

$1 - F(6) = 1 - (1 - e^{-6}) = \boxed{e^{-6} = .0025 = \frac{1}{e^6}}$

.0024788

hardway

or $f(y) = e^{-y}$ & $\int_6^\infty e^{-y}\,dy = -e^{-y}\Big|_6^\infty = 0 - (-e^{-6}) = e^{-6}$

5

-4 for f(y) 6