like 480 Exam 2 (which will have poisson processes)

2015

Math 483 Exam 3, Fall 2001 Name____________________

y	-2	-1	1	2
p(y)	7/20	3/20	6/20	4/20
y^2	4	1	1	4

← complete

10 5 e 1) Let the discrete random variable Y have a probability function given by the table above. Find the probability function of $U = Y^2$.

u	1	4
$p_U(u)$	$\frac{9}{20}$	$\frac{11}{20}$

20 6 2) Suppose that $Y_1, ..., Y_n$ are independent random variables where $E(Y_i) = r_i/p$, $V(Y_i) = \frac{r_i(1-p)}{p^2}$ and the moment generating function of Y_i is

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$$m_{Y_i}(t) = [\frac{pe^t}{1-(1-p)e^t}]^{r_i}$$

for any real t. Let $U = \sum_{i=1}^{n} Y_i$.

5 a) Find E(U). $= \sum_{i=1}^{n} E(Y_i) = \boxed{\frac{1}{p} \sum_{i=1}^{n} r_i}$

5 b) Find the variance V(U) of U. $= \sum_{i=1}^{n} V(Y_i) = \boxed{\frac{1-p}{p^2} \sum_{i=1}^{n} r_i}$

10 c) Find the moment generating function $m_U(t)$ of U.

$$= \prod_{i=1}^{n} m_{Y_i}(t) = \prod_{i=1}^{n} \left[\frac{pe^t}{1-(1-p)e^t}\right]^{r_i} = \boxed{\left[\frac{pe^t}{1-(1-p)e^t}\right]^{\sum_{i=1}^{n} r_i}}$$

not simplified −2 or 3

nr_i −3

$h(y) = -\log(y)$
$h(0) = \infty$, $h(1) = 0$

$\log(y)$ is increasing from $-\infty$ to 0 on $y \in (0,1]$

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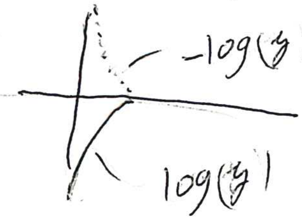
3) Suppose that Y is a random variable with pdf

so $-\log Y$ goes from 0 to ∞



$$f(y) = \frac{1}{\lambda} y^{\frac{1}{\lambda}-1},$$

~~10~~ 1
9

where $0 < y \leq 1$ and $\lambda > 0$. Let $U = -\log(Y)$ and find the pdf of U. Do not forget to include the support of U. Recall that $\log(y)$ is the natural logarithm in this class.

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$U = -\log(Y)$ so $\log Y = -U$ or $\underbrace{Y = e^{-U} = h^{-1}(U)}$ ~4

$\therefore \left| \frac{d h^{-1}(u)}{du} \right| = |-e^{-u}| = e^{-u}$ and

or at least -5

support wrong -3

$f_U(u) = f_Y(h^{-1}(u)) \left| \frac{d h^{-1}(u)}{du} \right| =$

$\frac{1}{\lambda} (e^{-u})^{\frac{1}{\lambda}-1} e^{-u} = \frac{1}{\lambda} e^{-u/\lambda} (e^{-u})^{-1} e^{-u}$

$= \boxed{\frac{1}{\lambda} e^{-u/\lambda}, \ u > 0}$

$0 < e^{-u} \leq 1$

$-\infty = \log 0 < -u \leq \log 1 = 0$

$\infty > u \geq 0$

e

5

4) Assume that $\overline{Y}$ is computed from a random sample of size $n = 4$ drawn from a highly skewed population with mean $\mu = 12$ and standard deviation $\sigma = 1.6$. If possible, find $P(11.8 < \overline{Y})$.

$\boxed{\text{not possible, } n \text{ too small for CLT}}$

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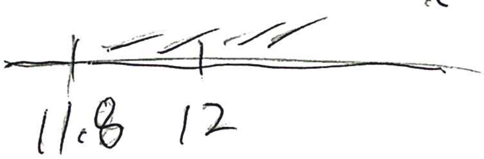
or -4

5

5) Assume that $\overline{Y}$ is computed from a random sample of size $n = 4$ drawn from a normal population with mean $\mu = 12$ and standard deviation $\sigma = 1.6$. If possible, find $P(11.8 < \overline{Y})$.

$\mu_{\bar{X}} = \mu = 12$ $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{4}} = 0.8$



$\frac{11.8 - 12}{0.8} = -0.25$

-3

prob $= 1 - P(Z > 0.25) = 1 - .4013 = \boxed{.5987}$

$.05$
$.201 \ .403$

$E Y_1 = \sum\sum y_1 p(y_1, y_2) = 0\frac{1}{2} + 0(0) + 1(\frac{1}{2}) + 1(0) = \frac{1}{2}$

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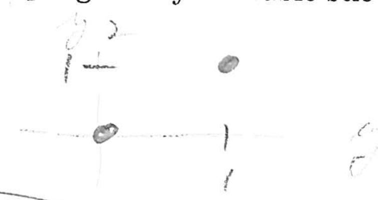
6) Suppose that the joint probability function of Y_1 and Y_2 is given by the table below.

$p(y_1, y_2)$		y_1 = 0	y_1 = 1	
y_2	0	1/2	0	$\frac{1}{2}$
	1	0	1/2	$\frac{1}{2}$
		$\frac{1}{2}$	$\frac{1}{2}$	



$\overline{25}$

5 e a) Are Y_1 and Y_2 independent? Explain.

No, support is not a cross product

or $p(0,0) = \frac{1}{2} \neq \frac{1}{2}\frac{1}{2} = \frac{1}{4} = p_{Y_1}(0)\, p_{Y_2}(0)$

$p(1,1)$ $\qquad p_{Y_1}(1)\, p_{Y_2}(1)$

e 5 b) Find the marginal probability function $p_{Y_2}(y_2)$ for Y_2.

y_2	0	1
$p_{Y_2}(y_2)$	$\frac{1}{2}$	$\frac{1}{2}$

−2 if they have

e 5 c) Find $E(Y_1)$.

$\frac{1}{2}(0) + \frac{1}{2}(1) = \boxed{\frac{1}{2}}$

e 5 d) Find $E(Y_2)$.

$\frac{1}{2}(0) + \frac{1}{2}(1) = \boxed{\frac{1}{2}}$ or by symmetry

5 e) Find $\text{Cov}(Y_1, Y_2)$. $E Y_1 Y_2 = \sum\sum y_1 y_2 p(y_1, y_2)$

$= 0(0)\frac{1}{2} + 0(1)\,0 + 1(0)\,0 + 1(1)\frac{1}{2} = \frac{1}{2}$

So $\text{cov}(Y_1, Y_2) = E Y_1 Y_2 - E Y_1 E Y_2 = \frac{1}{2} - \frac{1}{2}\frac{1}{2} = \boxed{\frac{1}{4}}$

or $E Y_1 = \int_0 \int_0 \frac{1}{4} y_1 \, dy_1 \, dy_2 = \int_0 \frac{1}{4} \frac{y_1^2}{2} \Big|_0 dy_2 = \int_0 \frac{1}{2} dy_2 = \frac{2}{2}$

or $E Y_1 = \int_0^2 \int_0^2 \frac{1}{4} y_1 \, dy_2 \, dy_1 = \int_0^2 \frac{1}{4} y_1 \, y_2 \Big|_0^2 dy_1 = \int_0^2 \frac{1}{2} y_1 \, dy_1$

$= \frac{y_1^2}{4} \Big|_0^1 = 1$

7) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$\overline{25}$

$$f(y_1, y_2) = \begin{cases} c, & \text{if } 0 \le y_1 \le 2, 0 \le y_2 \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

5 a) Find c. $\frac{1}{4}$ by geometry $\int_0^2 \int_0^2 c \, dy_2 \, dy_1 = \int_0^2 c \, y_2 \Big|_0^2 dy_1 = \int_0^2 2c \, dy_1$

$= 2c \, y_1 \Big|_0^2 = 4c$

$\int_0^2 \int_0^2 c \, dy_1 \, dy_2 = \int_0^2 c \, y_1 \Big|_0^2 dy_2 = \int_0^2 2c \, dy_2$

$= 2c \, y_2 \Big|_0^2 = 4c = 1$ or $\boxed{c = \frac{1}{4}}$

5 b) Find the marginal pdf of Y_1. Include the support. ← or −1

$f_{Y_1}(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) \, dy_2 = \int_0^2 \frac{1}{4} dy_2 = \frac{1}{4} y_2 \Big|_0^2 = \frac{1}{2}$

$\boxed{f_{Y_1}(y_1) = \frac{1}{2}, \quad 0 \le y_1 \le 2}$

5 c) Find $E(Y_1)$.

$\int_0^2 y_1 f_{Y_1}(y_1) \, dy_1 = \int_0^2 \frac{1}{2} y_1 \, dy_1 = \frac{y_1^2}{4} \Big|_0^2 = \frac{4}{4} = \boxed{1}$

5 d) Find $V(Y_1)$. $E Y_1^2 = \int_0^2 \frac{1}{2} y_1^2 \, dy_1 = \frac{1}{2} \frac{y_1^3}{3} \Big|_0^2 = \frac{8}{6} = \frac{4}{3}$

So $V(Y_1) = E Y_1^2 - (E Y_1)^2 = \frac{4}{3} - 1 = \boxed{\frac{1}{3}}$

→ 5 e) Are Y_1 and Y_2 independent? Explain.

Yes,

$f(Y_1, Y_2) = \sqrt{c} \, \sqrt{c} = g(Y_1) \, h(Y_2)$

on cross product support

$= \frac{1}{2} \, \frac{1}{2} = f_{Y_1}(y_1) \, f_{Y_2}(y_2)$

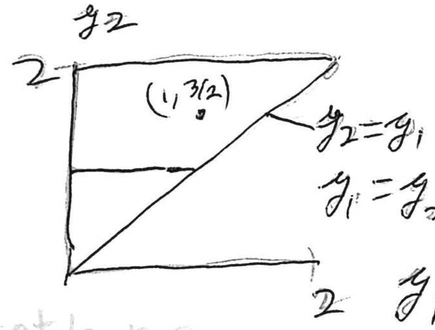
$E Y_1 Y_2 = \frac{1}{4} \int_0^2 \int_0^2 y_1 y_2 \, dy_1 \, dy_2 = \frac{1}{4} \left[\frac{y_1^2}{2} \Big|_0^2 \right] \left[\frac{y_2^2}{2} \Big|_0^2 \right] = \frac{1}{4} \cdot 2 \cdot 2 = 1$ $\text{Cov}(Y_1, Y_2) = 1 - 1(1) = 0$

8) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{2} & \text{if } \overline{0 \leq y_1 \leq y_2 \leq 2} \\ 0, & \text{otherwise.} \end{cases}$$

$1 \leq \frac{3}{2}$

Find the marginal pdf of Y_2. Include the support.



or at least -3

$$f_{Y_2}(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1 = \int_0^{y_2} \frac{1}{2}\, dy_1 = \frac{1}{2} y_1 \Big|_0^{y_2}$$

$$= \boxed{\frac{y_2}{2}, \; 0 < y_2 < 2}$$

or -2

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