2.5 too long Omit normal approx to binomial

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1) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{\theta}{y^{\theta+1}} \text{ where } y > 1$$

and $\theta > 1$.

$$E(Y) = \frac{\theta}{\theta - 1}.$$

Find the method of moments estimator for θ.

$\frac{\theta}{\theta-1} \stackrel{set}{=} \bar{Y}$ or $\theta = \theta\bar{Y} - \bar{Y}$

or $\theta\bar{Y} - \theta = \bar{Y}$ or $\theta(\bar{Y}-1) = \bar{Y}$

or $\boxed{\hat{\theta} = \frac{\bar{Y}}{\bar{Y}-1}} = \frac{-\bar{Y}}{1-\bar{Y}} = \frac{1}{-\frac{1}{\bar{Y}}+1}$

10 usually →

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2) A botanist measured the heights of a random sample of 15 seedlings and obtained a mean of 72.5 cm and a standard deviation of 4.5 cm. If the heights are from a normal distribution, find a 99% confidence interval for the population mean seedling height, if possible.

$t_{.005}$	$df = n-1$
2.977	14
99%	

$\bar{Y} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 72.5 \pm 2.977 \frac{4.5}{\sqrt{15}}$

2.576
−6

3.4590

$= 72.5 \pm 3.45896$

$= \boxed{[69.0410, 75.9590]}$

10

(69.0411, 75.9589)

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3) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{2y}{\theta^2}, \quad 0 < y < \theta.$$

Then $E(Y_i) = 2\theta/3$ and $V(Y_i) = \theta^2/18$. Let $T = c\overline{Y}$ be an estimator of θ where c is a constant.

a) Find the bias of T as a function of c and n. (Hint: $E(c\overline{Y}) = cE(Y_i)$ and the bias $B(T) = E(T) - \theta$.)

$$B(T) = Ec\overline{Y} - \theta = \boxed{\frac{c2\theta}{3} - \theta = \left(\frac{2}{3}c - 1\right)\theta}$$

b) Find the mean square error of T as a function of c and n. (Hint: $V(T) = \frac{c^2}{n^2}\sum_{i=1}^n V(Y_i)$ and $MSE(T) = V(T) + [B(T)]^2$.)

$$V(T) = \frac{c^2\theta^2}{18n} \quad \text{so}$$

usually −5

$$\boxed{MSE(T) = \frac{c^2\theta^2}{18n} + \left(\frac{2}{3}c\theta - \theta\right)^2}$$

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4) Suppose that the probability that a patient recovers from a certain blood disease is 0.4. Find the approximate probability that at least 35 of the next 100 patients who contract this disease survive.

E4d12

$9\frac{p}{1-p}$

$= 9\frac{1-p}{p} =$

$9\frac{.6}{.4} = 13.5 < n = 100$

Let Y = # of next 100 that survive

4.80
E3d20

$Y \sim bin(n=100, p=.4)$ want $P(Y \geq 35)$

$\approx P(X \geq 34.5)$ where $\mu_x = np = 40$

34.5 35

$\sigma_x = \sqrt{np(1-p)} = \sqrt{100(.4).6} = \sqrt{24} = 4.8990$

−5

$z = \frac{34.5 - 40}{4.8990} = -1.12$ ← minor −3

1.1 | .02
.1314

35 or 35.5

$= 1 -$ prob $= 1 - .1314 = \boxed{.8686}$

−1.12 1.12

10

wrong formula, at least →

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5) A company manufactures a large number of ball point pens each day. A random sample of 500 pens is selected from the day's production and 50 are defective. Find a 95% confidence interval for the proportion of defective pens produced by the manufacturer. (Hint: $\hat{p} = 50/500$.)

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .1 \pm 1.96\sqrt{\frac{.1(.9)}{500}}$$

E4d12

$$= .1 \pm 1.96\sqrt{.00018} = .1 \pm 1.96(.013416)$$

$$= .1 \pm .0263 = \boxed{[0.0737, 0.1263]}$$

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6) Suppose that Y is a random variable with pdf

$$f(y) = \frac{e^{-y}}{(1+e^{-y})^2},$$

$h(y) = e^{-y/\beta}$

Q8 problem

$h(-\infty) = \infty$, $h(\infty) = 0$

where $-\infty < y < \infty$. Let $U = e^{-Y/\beta}$ where $\beta > 1$. Find the pdf of U. Do not forget to include the support of U. (Hint: $e^{\beta \log(u)} = u^\beta$).

$$\log u = \frac{-y}{\beta}, \quad -y = \beta \log u, \quad y = -\beta \log(u) = h^{-1}(u)$$

$$\left|\frac{dh^{-1}(u)}{d\beta}\right| = \left|\frac{-\beta}{u}\right| = \frac{\beta}{u}$$

<0 at least -4

$$f_U(u) = f_Y(h^{-1}(u))\left|\frac{dh^{-1}(u)}{d\beta}\right| = \frac{e^{-(-\beta\log(u))}}{(1+e^{-(-\beta \log(u))})^2}\frac{\beta}{u}$$

$$= \frac{\beta}{u}\frac{e^{\beta \log u}}{(1+e^{\beta \log u})^2} = \frac{\beta}{u}\frac{u^\beta}{(1+u^\beta)^2} = \boxed{\frac{\beta u^{\beta-1}}{(1+u^\beta)^2}, \; u>0}$$

simplify or -1

= 4

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7) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \beta e^{-\beta(y-3)},$$

where $y > 3$ and $\beta > 0$.

a) Find the maximum likelihood estimator of β. Be sure to show why your answer is the global maximizer of the log likelihood.

8 got it

$$L(\beta) = \prod_{i=1}^{n} \beta e^{-\beta(y_i-3)} = \beta^n e^{-\beta \sum_{i=1}^{n}(y_i-3)}$$

$$\log(L(\beta)) = n\log(\beta) - \beta\sum_{i=1}^{n}(y_i-3)$$

$$\frac{d\log(L(\beta))}{d\beta} = \frac{n}{\beta} - \sum_{i=1}^{n}(Y_i-3) \overset{set}{=} 0$$

$$\text{or} \quad n = \beta\sum_{i=1}^{n}(y_i-3) \quad \text{or}$$

$$\boxed{\hat{\beta} = \frac{n}{\sum_{i=1}^{n}(Y_i-3)} = \frac{n}{\sum Y_i - 3n} = \frac{1}{\bar{Y}-3}}$$

od ~ -3

unique

$$\frac{d^2}{d\beta^2}\log(L(\beta)) = \frac{d}{d\beta}\left[\frac{n}{\beta} - \sum(Y_i-2)\right] = \frac{-n}{\beta^2} < 0$$

so $\hat{\beta}$ is the MLE

$$\frac{d^2}{d\beta^2}\log L(\beta)\Big|_{\beta=\hat{\beta}} = \frac{-n}{\left(\frac{n}{\sum Y_i-3}\right)^2} = \frac{-(\sum(Y_i-3))^2}{n} < 0$$

or -3 or -4

$= -\frac{n}{\beta^2} < 0$

b) What is the maximum likelihood estimator of $\log(\beta)$? Explain.

$$\boxed{\log(\hat{\beta}) = \log\left(\frac{n}{\sum(Y_i-3)}\right) = \log\left(\frac{n}{\sum Y_i - 3n}\right) = \log\left(\frac{1}{\bar{Y}-3}\right)}$$

by invariance

3

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8) The recommended daily allowance (RDA) for zinc for males over 50 is 15 mg/day. A researcher believes that older males may be taking less than the RDA of zinc. Suppose $n = 26$, $\bar{x} = 10.3$, $s = 6.43$ and the CLT applies. Do a 4 step test of hypotheses.

25 12.3

$H_0\ \mu = 15 \qquad H_A\ \mu < 15$

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{12.3 - 15}{6.43/\sqrt{25}} = \frac{-2.7}{1.286} = -2.10$$

$df = n - 1 =$

d		
24	-2.492	-2.064
left tail	.01	.025

$.01 < pval < .025$

reject Ho ← wrong −5

older men are getting less than the RDA } NO GENERALIZE −5

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