2015

Math 483   Quiz 10 2011   Name____________________

1) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{3\theta^3}{y^4} \quad \text{where} \quad y > \theta > 0.$$

a) Find E(Y). $= \int y f(y) dy = \int_\theta^\infty y \frac{3\theta^3}{y^4} dy = \int_\theta^\infty 3\theta^3 y^{-3} dy$

$= 3\theta^3 \frac{y^{-2}}{-2} \Big|_\theta^\infty = -\frac{3}{2}\theta^3 [0 - \theta^{-2}] = \boxed{\frac{3\theta}{2}}$

b) Find the method of moments estimator for θ.

$\frac{3\theta}{2} \overset{set}{=} \bar{Y}$ or $\boxed{\hat{\theta} = \frac{2\bar{Y}}{3}}$

$\overline{40}$

2) A student takes the temperatures of 6 randomly selected people. Assume that the temperatures follow a normal distribution, that the sample mean of the temperatures was 98.3 and that the sample standard deviation of the temperatures was 0.3127. Test the claim that the mean human temperature is different from 98.6 degrees.

i) $H_0\ \mu = 98.6 \quad H_A\ \mu \neq 98.6$

ii) $t_0 = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} = \frac{98.3 - 98.6}{0.3127/\sqrt{6}} = -2.35$

iii)

df=n-1		
5	-2.571	-2.015
two tail	.05	.10

$\alpha = .05 < \text{pval} < .1$

iv) Fail to reject H_0, the mean temp is 98.6

(or not enough evidence to conclude that the mean temp differs from 98.6)

$\overline{20}$

3) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{\alpha \ \theta^\alpha}{y^{\alpha+1}}$$

where $0 < \theta \le y$, θ is known, and $\alpha > 0$.

a) Find the maximum likelihood estimator of α. (Make sure that you prove that your answer is the MLE.)

$$L(\alpha) = \prod_{i=1}^{n} f(y_i) = \frac{\alpha^n \theta^{n\alpha}}{\prod_{i=1}^{n} y_i^{\alpha+1}} \quad \Leftarrow \text{or} \Rightarrow$$

$$\log L(\alpha) = n \log \alpha + n\alpha \log \theta - (\alpha+1) \sum_{i=1}^{n} \log(y_i)$$

$$\frac{d \log L(\alpha)}{d\alpha} = \frac{n}{\alpha} + n \log\theta - \sum_{i=1}^{n} \log(y_i) \overset{set}{=} 0$$

or $n + \alpha\left[n \log\theta - \sum \log(y_i)\right] = 0$ or

$$\hat{\alpha} = \frac{-n}{n\log\theta - \sum_{i=1}^{n} \log(Y_i)} = \frac{n}{\sum_{i=1}^{n} \log Y_i - n\log(\theta)}, \text{ unique}$$

$$= \frac{-1}{\log\theta - \frac{1}{n}\sum \log Y}$$

$$\underbrace{\frac{d^2 \log L(\alpha)}{d\alpha^2} = \frac{-n}{\alpha^2}}_{\text{or } -6} < 0 \text{ so } \hat{\alpha} \text{ is the MLE}$$

b) What is the maximum likelihood estimator of $1/\alpha$? Explain.

$$\widehat{\left(\frac{1}{\alpha}\right)} = \frac{\sum_{i=1}^{n} \log Y_i - n\log\theta}{n} = \frac{n\log(\theta) - \sum_{i=1}^{n} \log Y_i}{-n}$$

by invariance

or ~~[illegible]~~ -5

$= \frac{1}{n}\sum_{i=1}^{n} \log Y_i - \log\theta$

$\frac{90+40}{40}$

$\overline{40}$