Math 483 Quiz 3 fall 2005 Name____________________

1) Suppose that the moment generating function (mgf) of a random variable Y is

$$m(t) = \exp[\lambda(e^t - 1)]$$ Poisson(λ) mgf

where $\lambda > 0$ is a **known constant.**

a) Find the derivative $m'(t)$. chain rule

$$\boxed{\exp(\lambda(e^t-1))\,[\lambda e^t]}$$

$$= \lambda e^{\lambda e^t + t - \lambda} = \lambda e^{t + \lambda(e^t - 1)}$$

b) Find $E(Y)$. $m'(0) = \exp(0)\,\lambda e^0 = \boxed{\lambda}$

c) Find the second derivative $m''(t)$. $= \lambda e^{t+\lambda(e^t-1)}[1+\lambda e^t]$

$$\boxed{\exp[\lambda(e^t-1)][\lambda e^t]^2 + \exp[\lambda(e^t-1)]\lambda e^t}$$

$$= [(\lambda e^t)^2 + \lambda e^t]\exp[\lambda(e^t-1)] = \lambda e^t(1+\lambda e^t)\exp[\lambda(e^t-1)]$$

d) Find $E(Y^2)$. $m''(0) = \exp(0)\,\lambda^2 + \exp(0)\,\lambda e^0$

$$= \boxed{\lambda^2 + \lambda}$$

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$\ln[m'(t)] = \ln\lambda + t + \lambda(e^t-1)$

$\frac{d}{dt}\ln m'(t) = \frac{m''(t)}{m'(t)} = 1 + \lambda e^t$, $m''(t) = (1+\lambda e^t)\lambda e^t \exp[\lambda(e^t-1)]$

y	0	1	2	3
p(y)	0.08	0.32	0.42	0.18

2) The table above displays the probability distribution for a discrete random variable Y. Find the moment generating function of Y. (Hint: $m(t) = \sum e^{ty}p(y)$.)

$$\boxed{e^{t0}(.08) + e^{t1}(.32) + e^{t2}(.42) + e^{t3}(.18) = 0.08 + 0.32e^{t} + 0.42e^{2t} + 0.18e^{3t}}$$

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3) Suppose Y is binomial with $n = 25$ and $p = 0.9$.

a) Find $E(Y)$.

$$np = 25(.9) = \boxed{22.5}$$

b) Find $P(Y > 23)$. $= P(24) + P(25) =$

$$\binom{25}{24}(.9)^{24}(.1) + \binom{25}{25}(.9)^{25}(.1)^{0}$$

$$= 25(.9)^{24}(.1) + (.9)^{25} = .1994 + .0718$$

$$= \boxed{0.2712}$$

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4) Suppose Y is Poisson with $\lambda = 9$.

a) Find $E(Y)$.

$$= \lambda = \boxed{9}$$

or 9

b) Find $P(Y = 5)$.

$$P(y) = \frac{\lambda^{y} e^{-\lambda}}{y!}, \quad P(5) = \frac{9^{5} e^{-9}}{5!} = \boxed{0.0607}$$

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