

Fall 2015
Fall 2012

quotient rule $\frac{0 - (-\mu e^t)}{[1 + \mu(1 - e^t)]^2}$

1) Suppose that the moment generating function (mgf) of a random variable Y is

$$m(t) = \frac{1}{1 + \mu(1 - e^t)}$$

GNB($\mu, \lambda=1$)

where $\mu > 0$ is a known constant and $t < -\log(\mu/(1 + \mu))$. Using the mgf $m(t)$, find $E(Y)$.

$$m'(t) = \frac{d}{dt} [1 + \mu(1 - e^t)]^{-1} = - [1 + \mu(1 - e^t)]^{-2} (-\mu e^t)$$

$$= \mu e^t [1 + \mu(1 - e^t)]^{-2}$$

wrong deriv
-505-6

$$E(Y) = m'(0) = \mu(1) [1 + \mu(1 - 1)]^{-2} = \mu$$

quotient rule $m'(t) = \frac{0 - (-\mu e^t)}{[1 + \mu(1 - e^t)]^2} = \frac{\mu e^t}{[1 + \mu(1 - e^t)]^2}$

2) Suppose that Y is a random variable with moment generating function $m(t) = \frac{1}{8}e^{3t} + \frac{4}{8}e^{5t} + \frac{2}{8}e^{7t} + \frac{1}{8}e^{11t}$. Find $E(Y)$.

$$m'(t) = \frac{1}{8}(3)e^{3t} + \frac{4}{8}(5)e^{5t} + \frac{2}{8}(7)e^{7t} + \frac{1}{8}(11)e^{11t}$$

$$m'(0) = E(Y) = \frac{3}{8} + \frac{20}{8} + \frac{14}{8} + \frac{11}{8} = \frac{48}{8} = 6$$

3) Suppose that Y is a random variable with distribution function

$$F(y) = 1 - \frac{1}{1 + y^2}$$

log logistic $\phi=1$
 $\tau=2$

for $y > 0$ and that $F(y) = 0$ otherwise. Find the probability density function (pdf) $f(y)$ of Y .

$$f(y) = F'(y) = \frac{d}{dy} - (1 + y^2)^{-1} = (-1) (- (1 + y^2)^{-2}) 2y$$

negative
-9

quotient rule $\frac{0 - (-1) 2y}{(1 + y^2)^2} = \frac{2y}{(1 + y^2)^2}$

$$= \begin{cases} \frac{2y}{(1 + y^2)^2}, & y > 0 \\ 0, & \text{else} \end{cases}$$

4) Suppose that the probability density function for a random variable Y is given by

$$f(y) = \begin{cases} cy, & \text{if } 0 \leq y \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c .

$$\int_0^4 cy \, dy = \left. \frac{cy^2}{2} \right|_0^4 = 8c \stackrel{\text{set}}{=} 1$$

$$c = \frac{1}{8} = 0.125$$

b) Find $E(Y)$. $= \int_0^4 y \cdot \frac{1}{8} y \, dy = \int_0^4 \frac{1}{8} y^2 \, dy = \frac{1}{8} \left. \frac{y^3}{3} \right|_0^4$

$$= \frac{64}{8(3)} = \frac{8}{3} = 2.6667$$

5) The lengths of human pregnancy X follow a normal distribution with mean $\mu = 266$ and standard deviation $\sigma = 16$ days.

a) What is the length of pregnancy such that 90% of pregnancies are longer?

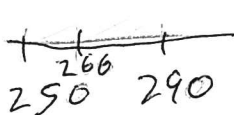


1.2	0.08
1.003	

$$z_0 = -1.28$$

$$x_0 = \mu + \sigma z_0 = 266 + 16(-1.28) = 245.52$$

b) Find the probability that X will be between 250 and 290.



$$\frac{250 - 266}{16} = -1.00, \quad \frac{290 - 266}{16} = 1.50$$

$$P(250 < X < 290) = 1 - 0.1587 - 0.0668 = 0.7745$$

	0.00
1.0	0.1587
1.5	0.0668