2015

Math 483 Quiz 5 2010 Name____________________

1) Suppose that the joint probability function $p(y_1, y_2)$ of Y_1 and Y_2 is and is tabled as shown.

$p(y_1, y_2)$		y_1 = 1	2	3	4	
y_2	1	0.0878	0.2100	0.1715	0.2059	.6752
	2	0.0121	0.0150	0.0097	0.0071	.0439 ←
	3	0.0171	0.0452	0.0379	0.0104	.1106
	4	0.0257	0.0513	0.0439	0.0494	.1703

a) Find the marginal probability function $p_{Y_2}(y_2)$ for Y_2.

y_2	1	2	3	4
$p_{Y_2}(y_2)$	.6752	.0439	.1106	.1703

b) Find the conditional probability function $p(y_1|y_2)$ of Y_1 given $Y_2 = 2$.

$$= \frac{p(y_1, y_2)}{p_{Y_2}(y_2)} = \frac{p(y_1, 2)}{.0439}$$

y_1	1	2	3	4
$p(y_1\|2)$	.2756	.3417	.2210	.1617

easy way: fix $Y_2 = 2$ on table: $\frac{.0121}{.0439}$ $\frac{.0150}{.0439}$ $\frac{.0097}{.0439}$ $\frac{.0071}{.0439}$

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2) Suppose that IQ scores X follow a normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 15$.

a) What is the IQ score such that 2% of IQ scores are higher.

	.05	.06
2.0	.0202	.0197

$z_0 = 2.05$

$$x_0 = \mu + \sigma z_0 = 100 + 2.05(15) = \boxed{130.75}$$

b) Find the probability that X will be less than 75.

$$z = \frac{75 - 100}{15} = -1.67$$

75 100

	.07
1.6	.0475

$$P(X < 75) = P(Z > 1.67) = \boxed{0.0475}$$

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3) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} c, & \text{if } 0 \le y_1 \le 4, 0 \le y_2 \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c. $\boxed{\frac{1}{12}}$ by geometry

or $c\int_0^4 \int_0^3 dy_2\, dy_1 = \int_0^4 c\left[y_2 \big|_0^3\right] dy_1 = \int_0^4 3c\, dy_1 = 3c\, y_1 \big|_0^4 = 12c = 1$

or $c\int_0^3 \int_0^4 dy_1\, dy_2 = \int_0^3 c\left[y_1 \big|_0^4\right] dy_2 = \int_0^3 4c\, dy_2 = 4c\, y_2 \big|_0^3 = 12c = 1$

b) Find the marginal pdf of Y_1. Include the support.

$f_{Y_1}(y_1) = \int_0^3 f(y_1, y_2)\, dy_2 = \int_0^3 \frac{1}{12} dy_2 = \frac{1}{12} y_2 \big|_0^3 = \frac{3}{12}$

$= \boxed{\frac{1}{4},\ 0 < y_1 < 4}$

or -4

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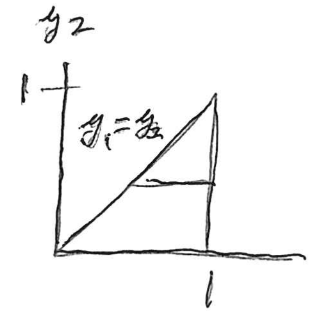
4) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 3y_1 & \text{if } 0 \le y_2 \le y_1 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$



Suppose that the marginal pdf of Y_2 is given by

later

$$f_{Y_2}(y_2) = \begin{cases} \frac{3}{2}(1 - y_2^2) & \text{if } 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional pdf of Y_1 given $Y_2 = y_2$, that is, find $f_{Y_1|Y_2=y_2}(y_1|y_2) \equiv f(y_1|y_2)$. Make sure you include the support of the conditional pdf. (Hint: sketch the support and draw in a horizontal line corresponding to a fixed value of y_2.)

$f_{Y_1|Y_2}(y_1|y_2) = \frac{f(y_1, y_2)}{f_{Y_2}(y_2)} = \frac{3y_1}{\frac{3}{2}(1 - y_2^2)} = \boxed{\frac{2y_1}{1 - y_2^2},\ y_2 \le y_1 \le 1}$

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