see 483 Q6d20

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Math 483 Quiz 6 Spring 2005 Name____________________

1) Suppose that the joint probability function of Y_1 and Y_2 is $p(y_1, y_2)$ is tabled below.

$p(y_1, y_2)$		y_1 = 0	1	2	$P(Y_2 = y_2)$
y_2	0	0/27	2/27	4/27	6/27
	1	1/27	3/27	5/27	9/27
	2	2/27	4/27	6/27	12/27
$P(Y_1 = y_1)$		$\frac{3}{27}$	$\frac{9}{27}$	$\frac{15}{27}$	

a) Are Y_1 and Y_2 independent? Explain.

$\boxed{\text{no support is not a cross product}}$

or $0 = p(0,0) \neq p_{Y_1}(0)\, p_{Y_2}(0) = \frac{3}{27}\frac{6}{27} > 0$

b) Find $E(Y_1)$. $= \sum y_1 p_{Y_1}(y_1) = 0\left(\frac{3}{27}\right) + 1\left(\frac{9}{27}\right) + 2\left(\frac{15}{27}\right) = \boxed{\frac{39}{27} = \frac{13}{9} \approx 1.444}$

or $EY_1 = \sum\sum y_1 p(y_1,y_2) = 0\frac{0}{27} + 0\frac{1}{27} + 0\frac{2}{27} + 1\frac{2}{27} + 1\frac{3}{27} + 1\frac{4}{27} + 2\frac{4}{27} + 2\frac{5}{27} + 2\frac{6}{27} = \frac{2+3+4+8+10+12}{27} = \frac{39}{27}$

hard way

c) Find $E(Y_2)$.

$= \sum y_2 p_{Y_2}(y_2) = 0\frac{6}{27} + 1\frac{9}{27} + 2\frac{12}{27} = \boxed{\frac{33}{27} = \frac{11}{9} \approx 1.222}$

or $EY_2 = 0\frac{0}{27} + 0\frac{2}{27} + 0\frac{4}{27} + 1\frac{1}{27} + 1\frac{3}{27} + 1\frac{5}{27} + 2\frac{2}{27} + 2\frac{4}{27} + 2\frac{6}{27} = \frac{1+3+5+4+8+12}{27} = \frac{33}{27}$

hard way

d) Find $\mathrm{Cov}(Y_1, Y_2)$.

$EY_1Y_2 = \sum\sum y_1 y_2\, p(y_1, y_2) = 0 + 0 + 0$
$+ 0 + 1(1)\frac{3}{27} + 1(2)\frac{5}{27}$
$+ 0 + 2(1)\frac{4}{27} + 2(2)\frac{6}{27} = \frac{3+10+8+24}{27} = \frac{45}{27} = \frac{5}{3}$ (1.667)

so $\mathrm{Cov}(Y_1,Y_2) = EY_1Y_2 - EY_1\,EY_2 = \frac{45}{27} - \frac{13}{9}\frac{11}{9} = -\frac{8}{81} = \boxed{-0.0988}$

Yes cross product support ?
Yes Cov$(Y_1, Y_2) = 0$

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-1 if support of Y_1, Y_2 ... OR -2

2) Suppose that the joint pdf of the random variables Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 4\ y_1 y_2, & \text{if } 0 \le y_1 \le 1, 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

or for $0 \le y_1 \le 1$, $0 \le y_2 \le 1$, 0 else

10 a) Are Y_1 and Y_2 independent? Explain.

Yes (support is a cross product and)

$f(y_1, y_2) = (2y_1)(2y_2) = h(y_1)\, g(y_2)$ on cross product support

or find $f_{Y_1}(y_1)$, $f_{Y_2}(y_2)$ and show $f(y_1, y_2) = f_{Y_1}(y_1)\, f_{Y_2}(y_2)$

10 b) Find $E(Y_1)$. $f_{Y_1}(y_1) = \int_0^1 4\, y_1 y_2\, dy_2 = 4\, y_1 \frac{y_2^2}{2}\Big|_0^1 = 2y_1,\ 0 \le y_1 \le 1$

so $EY_1 = \int_0^1 y_1\ 2y_1\, dy_1 = \int_0^1 2y_1^2\, dy_1 = 2\frac{y_1^3}{3}\Big|_0^1 = \boxed{\frac{2}{3}}$

or $EY_1 = \int_0^1 \int_0^1 4 y_1^2 y_2\, dy_2\, dy_1 = \int_0^1 4 y_1^2 \frac{y_2^2}{2}\Big|_0^1 dy_1 = \int_0^1 2y_1^2\, dy_1 = 2\frac{y_1^3}{3}\Big|_0^1 = \frac{2}{3}$

hard ways

or $EY_1 = \int_0^1 \int_0^1 4 y_1^2 y_2\, dy_1\, dy_2 = \int_0^1 4 y_2 \frac{y_1^3}{3}\Big|_0^1 dy_2 = \int_0^1 \frac{4}{3} y_2\, dy_2 = \frac{4}{3}\frac{y_2^2}{2}\Big|_0^1 = \frac{2}{3}$

10 c) Find $E(Y_2)$.

$\frac{2}{3}$ by symmetry or $f_{Y_2}(y_2) = \int_0^1 4 y_1 y_2\, dy_1 = 4 y_2 \frac{y_1^2}{2}\Big|_0^1 = 2y_2,\ 0 < y_2 < 1$

$EY_2 = \int_0^1 y_2\, 2y_2\, dy_2 = 2\frac{y_2^3}{3}\Big|_0^1 = \frac{2}{3}$

too many students did it the hard way

— hard way

10 d) Find Cov(Y_1, Y_2). $\boxed{0 \text{ by ind}}$

or $E Y_1 Y_2 = \int_0^1 \int_0^1 4 y_1^2 y_2^2\, dy_2\, dy_1 = \int_0^1 4 y_1^2 \frac{y_2^3}{3}\Big|_0^1 dy_1 = \int_0^1 \frac{4}{3} y_1^2\, dy_1 = \frac{4}{3}\frac{y_1^3}{3}\Big|_0^1 = \frac{4}{9}$

hard ways

or $E Y_1 Y_2 = \int_0^1 \int_0^1 4 y_1^2 y_2^2\, dy_1\, dy_2 = \int_0^1 4 \left[\frac{y_1^3}{3}\Big|_0^1\right] y_2^2\, dy_2 = \int_0^1 \frac{4}{3} y_2^2\, dy_2 = \frac{4}{3}\frac{y_2^3}{3} = \frac{4}{9}$

and $E Y_1 Y_2 - E Y_1 E Y_2 = \frac{4}{9} - \frac{2}{3}\frac{2}{3} = 0$

3) Suppose that X and Y are independent random variables and that $E(X^2) = 10$ while $E(Y^2) = 25$. Find $E(X^2\ Y^2)$.

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$E X^2\ E Y^2 = 10(25) = \boxed{250}$