$U = h(y) = 8y^3, \quad h(0) = 0, \quad h(1) = 8$

Math 483 Quiz 7 Fall 2006 Name____________________

1) Let Y be a random variable from a distribution with pdf 10

$$f(y) = 2y \text{ where } 0 < y < 1.$$

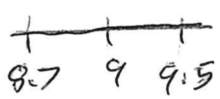
Let $U = 8Y^3$ and find the pdf of U using the method of transformations. Do not forget to include the support of U. $Y^3 = U/8$ so $Y = \frac{1}{2} U^{1/3} = h^{-1}(U)$

$\left|\frac{dh^{-1}(U)}{dU}\right| = \left|\frac{1}{3}\frac{1}{2} U^{-2/3}\right| = \frac{1}{6} U^{-2/3}$, $Y=0 \Rightarrow U=0$, $Y=1 \Rightarrow U=8$

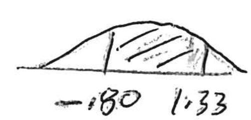
So $f_U(U) = f_Y(h^{-1}(U)) \left|\frac{dh^{-1}(U)}{dU}\right| = 2\left(\frac{1}{2} U^{1/3}\right) \frac{1}{6} U^{-2/3}$

$$\boxed{f_U(U) = \frac{1}{6} U^{-1/3} = \frac{1}{6}\frac{1}{U^{1/3}}, \quad 0 < U < 8}$$

2) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is approximately normal. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean $\overline{Y}$ of their reading times. Find the probability that $\overline{Y}$ is between 8.7 and 9.5 minutes. $\mu_{\overline{Y}} = 9$ $\quad \sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{16}} = 0.375 = 3/8$



$\frac{8.7-9}{.375} = -0.80$, $\frac{9.5-9}{.375} = 1.33$



$= 1 - .2119 - .0918 = \underline{.6963}$

3) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is highly skewed. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean $\overline{Y}$ of their reading times. Find the probability that $\overline{Y}$ is between 8.7 and 9.5 minutes.

$\boxed{\text{not possible}}$ highly skewed $n = 16 \leq 30$

y	−3	−1	1	2
p(y)	3/20	7/20	6/20	4/20
y^2+y	6	0	2	6

4) Let the discrete random variable Y have a probability function given by the table above. Find the probability function of $U = Y^2 + Y$.

U	0	2	6
p(U)	$\frac{7}{20}$	$\frac{6}{20}$	$\frac{7}{20}$

5) Suppose that $Y_1, ..., Y_n$ are independent random variables where $E(Y_i) = V(Y_i) = \lambda_i$ and the moment generating function of Y_i is $m_{Y_i}(t) = \exp[(e^t - 1)\lambda_i]$ for any real t. Let $U = \sum_{i=1}^{n} Y_i$.

a) Find E(U). $= \sum E Y_i = \boxed{\sum_{i=1}^{n} \lambda_i}$

b) Find the variance V(U) of U. $= \sum_{i=1}^{n} V(Y_i) = \boxed{\sum_{i=1}^{n} \lambda_i}$

c) Find the moment generating function $m_U(t)$ of U.

$$= \prod_{i=1}^{n} m_{Y_i}(t) = \prod_{i=1}^{n} \exp\left[(e^t - 1)\lambda_i\right] = \boxed{\exp\left[(e^t - 1)\sum_{i=1}^{n} \lambda_i\right]}$$