

$$U = h(Y) = 8Y^3, \quad h(0) = 0, \quad h(1) = 8$$

Math 483

Quiz 7 Fall 2006

Name _____

1) Let Y be a random variable from a distribution with pdf

$$f(y) = 2y \quad \text{where } 0 < y < 1.$$

Let $U = 8Y^3$ and find the pdf of U using the method of transformations. Do not forget to include the support of U .

$$y^3 = u/8 \quad \text{so } y = \frac{1}{2} u^{1/3} = h^{-1}(u)$$

$$\left| \frac{dh^{-1}(u)}{du} \right| = \left| \frac{1}{3} \cdot \frac{1}{2} u^{-2/3} \right| = \frac{1}{6} u^{-2/3}, \quad y=0 \Rightarrow u=0, \quad y=1 \Rightarrow u=8$$

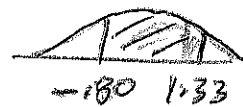
$$\text{So } f_U(u) = f_Y(h^{-1}(u)) \left| \frac{dh^{-1}(u)}{du} \right| = 2 \left(\frac{1}{2} u^{1/3} \right) \frac{1}{6} u^{-2/3}$$

$$f_U(u) = \frac{1}{6} u^{-1/3} = \frac{1}{6} \frac{1}{u^{1/3}}, \quad 0 < u < 8$$

2) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is approximately normal. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean \bar{Y} of their reading times. Find the probability that \bar{Y} is between 8.7 and 9.5 minutes.

$$\mu_{\bar{Y}} = 9 \quad \sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{16}} = 0.375 = 3/8$$

$$\frac{8.7-9}{0.375} = -0.80, \quad \frac{9.5-9}{0.375} = 1.33$$



$$= 1 - 0.2119 - 0.0918 = 0.6963$$

3) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is highly skewed. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean \bar{Y} of their reading times. Find the probability that \bar{Y} is between 8.7 and 9.5 minutes.

not possible

highly skewed
 $n=16 \leq 30$

old E3 1/2/3/4/5/1
old final 3/0/1

y	-3	-1	1	2
p(y)	3/20	7/20	6/20	4/20

y^2+y 6 0 2 6 ←

4) Let the discrete random variable Y have a probability function given by the table above. Find the probability function of $U = Y^2 + Y$.

U	0	2	6
p(u)	$\frac{7}{20}$	$\frac{6}{20}$	$\frac{7}{20}$

5) Suppose that Y_1, \dots, Y_n are independent random variables where $E(Y_i) = V(Y_i) = \lambda_i$ and the moment generating function of Y_i is $m_{Y_i}(t) = \exp[(e^t - 1)\lambda_i]$ for any real t . Let

$$U = \sum_{i=1}^n Y_i.$$

a) Find $E(U)$.

$$= \sum E Y_i =$$

$$\sum_{i=1}^n \lambda_i$$

b) Find the variance $V(U)$ of U .

$$= \sum_{i=1}^n V(Y_i) =$$

$$\sum_{i=1}^n \lambda_i$$

c) Find the moment generating function $m_U(t)$ of U .

$$= \prod_{i=1}^n m_{Y_i}(t) =$$

$$\prod_{i=1}^n \exp[(e^t - 1)\lambda_i] =$$

$$\exp[(e^t - 1) \sum_{i=1}^n \lambda_i]$$

$$\exp[(e^t - 1) \sum_{i=1}^n \lambda_i]$$

Note: If $\lambda \equiv \lambda$ get $n\lambda$, $n\lambda$ and $\exp[(e^t - 1)n\lambda]$
for a) b) and c)