$U = h(y) = 8y^3, \quad h(0) = 0, \quad h(1) = 8$

Math 483 Quiz 7 Fall ~~2006~~ 2015 Name____________________

1) Let Y be a random variable from a distribution with pdf 10

$$f(y) = 2y \text{ where } 0 < y < 1.$$

Let $U = 8Y^3$ and find the pdf of U using the method of transformations. Do not forget to include the support of U. $y^3 = u/8$ so $y = \frac{1}{2} u^{1/3} = h^{-1}(u)$

$$\left|\frac{dh^{-1}(u)}{du}\right| = \left|\frac{1}{3}\frac{1}{2} u^{-2/3}\right| = \frac{1}{6} u^{-2/3}, \quad y = 0 \Rightarrow u = 0, \; y = 1 \Rightarrow u = 8$$

$$\text{so } f_U(u) = f_Y(h^{-1}(u)) \left|\frac{dh^{-1}(u)}{du}\right| = 2\left(\frac{1}{2} u^{1/3}\right)\frac{1}{6} u^{-2/3}$$

$$f_U(u) = \frac{1}{6} u^{-1/3} = \frac{1}{6}\frac{1}{u^{1/3}}, \quad 0 < u < 8$$

2) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is approximately normal. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean $\overline{Y}$ of their reading times. Find the probability that $\overline{Y}$ is between 8.7 and 9.5 minutes. $\mu_{\overline{Y}} = 9 \quad \sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{16}} = 0.375 = 3/8$

8.7 9 9.5 $\quad \frac{8.7-9}{.375} = -0.80, \quad \frac{9.5-9}{.375} = 1.33 \quad$ $-.80$ 1.33

$$= 1 - .2119 - .0918 = .6963$$

3) According to the 1998 USA TODAY, adults between the ages of 18 and 24 spend an average of $\mu = 9$ minutes per day reading the newspaper. Assume that $\sigma = 1.5$ minutes and that the distribution for the reading times is highly skewed. Suppose that you obtain a simple random sample of 16 adults ages 18 to 24 and compute the sample mean $\overline{Y}$ of their reading times. Find the probability that $\overline{Y}$ is between 8.7 and 9.5 minutes.

$\boxed{\text{not possible}}$ highly skewed $n = 16 \leq 30$

OWED 1 | 2 | 3 | 4 | 5 |
010 final 5 | 6 |

y	-3	-1	1	2
p(y)	3/20	7/20	6/20	4/20
y^2+y	6	0	2	6

4) Let the discrete random variable Y have a probability function given by the table above. Find the probability function of $U = Y^2 + Y$.

U	0	2	6
$p(u)$	$\frac{7}{20}$	$\frac{6}{20}$	$\frac{7}{20}$

5) Suppose that $Y_1, ..., Y_n$ are independent random variables where $E(Y_i) = V(Y_i) = \lambda_i$ and the moment generating function of Y_i is $m_{Y_i}(t) = \exp[(e^t - 1)\lambda_i]$ for any real t. Let $U = \sum_{i=1}^{n} Y_i$.

a) Find E(U). $= \sum E\,Y_i = \boxed{\sum_{i=1}^{n} \lambda_i}$

b) Find the variance V(U) of U. $= \sum_{i=1}^{n} V(Y_i) = \boxed{\sum_{i=1}^{n} \lambda_i}$

c) Find the moment generating function $m_U(t)$ of U.

$$= \prod_{i=1}^{n} m_{Y_i}(t) = \prod_{i=1}^{n} \exp\left[(e^t-1)\lambda_i\right] = \boxed{\exp\left[(e^t-1)\sum_{i=1}^{n}\lambda_i\right]}$$

Note: If $\lambda_i \equiv \lambda$ get $n\lambda$, $n\lambda$ and $\exp[(e^t-1)n\lambda]$ for a) b) and c)