2015

Math 483   Quiz 8 Spring 2005   Name____________________

20 → 1) Suppose that Y is a random variable with pdf

$$f(y) = \frac{e^{-y}}{(1+e^{-y})^2},$$

$h(y) = e^{-y/\beta}$

$h(-\infty) = \infty,\ h(\infty) = 0$

where $-\infty < y < \infty$. Let $U = e^{-Y/\beta}$ where $\beta > 1$. Find the pdf of U. Do not forget to include the support of U. (Hint: $e^{\beta \log(u)} = u^\beta$).

$\log u = \frac{-Y}{\beta}$, $-Y = \beta \log u$ so $Y = -\beta \log(u) = h^{-1}(u)$

$\left|\frac{d h^{-1}(u)}{du}\right| = \left|\frac{-\beta}{u}\right| = \frac{\beta}{u}$. So $f_U(u) = f_Y(h^{-1}(u)) \left|\frac{d h^{-1}(u)}{du}\right|$

$$= \frac{e^{-(-\beta \log u)}}{[1+e^{-(-\beta \log u)}]^2} \frac{\beta}{u} = \frac{\beta}{u} \frac{e^{\beta \log u}}{[1+e^{\beta \log u}]^2}$$

← simplify or −1

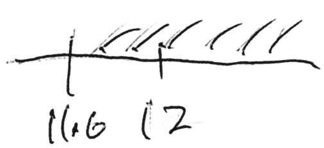
$$= \frac{\beta}{u} \frac{u^\beta}{(1+u^\beta)^2} = \boxed{\frac{\beta u^{\beta-1}}{(1+u^\beta)^2},\ u > 0}$$

or −8

20 2) Assume that $\overline{Y}$ is computed from a random sample of size $n = 16$ drawn from a normal population with mean $\mu = 12$ and standard deviation $\sigma = 2.7$. If possible, find $P(11.6 < \overline{Y})$.

$\mu_{\bar{Y}} = \mu = 12$ $\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{2.7}{\sqrt{16}} = \frac{2.7}{4} = 0.675$

or −15



$z = \frac{11.6 - 12}{0.675} = -0.59$

$= 1 - .2776 = \boxed{.7224}$

−8

−.6 and .7257 −3

	09
0.5	.2776

M480 HW11

3) Suppose that the probability that a patient recovers from a certain blood disease is 0.4. Find the approximate probability that at least 36 of the next 100 patients who contract this disease survive. (Hint: Let Y be the number of patients who recover, then Y is binomial($n = 100, p = 0.4$). Let X be a normal RV with mean $\mu = np$ and SD $\sigma = \sqrt{np(1-p)}$ and find $P(X \geq \cancel{34.5})$.) 35.5

$\mu_X = np = 40, \ \sigma_X = \sqrt{n\,p(1-p)} = \sqrt{100(.4)(.6)}$

$= \sqrt{24} = 4.8$

35.5 40

$z = \frac{35.5 - 40}{4.899} = -.92$

$-.91$

$-.92$

$= 1 - .1788 = \boxed{.8212}$

or .8186

-3

4) Let $Y_n, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{2y}{\theta^2}, \quad 0 < y < \theta.$$

Then $E(Y_i) = 2\theta/3$ and $V(Y_i) = \theta^2/18$. Let $T = c\overline{Y}$ be an estimator of θ where c is a constant.

a) Find the bias of T as a function of c and n. (Hint: $E(c\overline{Y}) = cE(Y_i)$ and the bias $B(T) = E(T) - \theta$.)

$B(T) = E(T) - \theta = Ec\overline{Y} - \theta =$

or -5 $\quad \boxed{c\frac{2\theta}{3} - \theta = \left(\frac{2}{3}c - 1\right)\theta}$ $\quad = \frac{2c\theta - 2\theta}{3}$

b) Find the mean square error of T as a function of c and n. (Hint: $V(T) = \frac{c^2}{n^2}\sum_{i=1}^n V(Y_i)$ and $MSE(T) = V(T) + [B(T)]^2$.)

$V(T) = \frac{c^2}{n^2} n \frac{\theta^2}{18} = \frac{c^2}{n} V(Y_i) = \frac{c^2\theta^2}{18n}$ $\quad -3$

or -5

So $\boxed{MSE(T) = \frac{c^2\theta^2}{18n} + \left(\frac{2}{3}c\theta - \theta\right)^2}$

5) Assume that a random sample from a normal distribution yields sample mean $\overline{Y} = 100$ and sample standard deviation $S = 36$. Find a 95% confidence interval for μ if the sample size $n = 7$.

df=60 or -11

-10

1.96

df = n−1	$t_{.025}$
6	2.447
	95%

$\overline{Y} \pm t_{\alpha/2}\frac{S}{\sqrt{n}} = 100 \pm 2.447\frac{36}{\sqrt{7}} = 100 \pm 33.296$

compute or -7

$= \boxed{[66.704, 133.296]}$