72 − 68 / 7.5127 = .532 and .7019

this → .6772 −3

−7

Math 483   Quiz 9   2010   Name____________________

2015

1) According to the Center for Disease control, 17% of Americans have high cholesterol. Suppose that a random sample of 400 Americans is selected. Find the approximate probability that the hospital will ask at most 72 had high cholesterol.

M480 E3022

$= \sqrt{56.44}$

$\mu_x = np = 400(.17) = 68$ $\sigma_x = \sqrt{np(1-p)} = \sqrt{400(.17).83} = 7.5127$

want $P(x < 72.5)$ 68 72.5 $z = \frac{72.5-68}{7.5127} = .60$

make this

−8

20

$= P(z < .60) = 1 - P(z > .60) = 1 - .2743 = \boxed{.7257}$

159 and $1 - .2776 = .7224$ −3

2) Let $Y_1, ..., Y_n$ be iid from a distribution with pdf $f(y) = \frac{1}{\lambda} \exp\left[(\frac{1}{\lambda} - 1)\log(y)\right]$ where $0 < y < 1$ and $\lambda > 0$. Then the likelihood $L(\lambda) = \prod_{i-1}^{n} f(y_i) = \frac{1}{\lambda^n} \exp\left[(\frac{1}{\lambda} - 1)\sum_{i=1}^{n} \log(y_i)\right]$, and the log likelihood

$$\log(L(\lambda)) = -n\log(\lambda) + (\frac{1}{\lambda} - 1)\sum_{i=1}^{n} \log(y_i).$$

lower

a) Find

$$\frac{d}{d\lambda}\log(L(\lambda)) \text{ and solve } \frac{\mathrm{d}}{\mathrm{d}\lambda}\log(\mathrm{L}(\lambda)) \overset{\text{set}}{=} 0 \text{ for } \hat{\lambda}.$$

$\frac{d}{d\lambda}\log L(\lambda) = \frac{-n}{\lambda} - \frac{1}{\lambda^2}\sum \log y_i \overset{set}{=} 0$ or $-n\lambda - \sum \log y_i = 0$

or $n\lambda = -\sum \log y_i$ or $\boxed{\hat{\lambda} = \frac{-1}{n}\sum_{i=1}^{n} \log Y_i}$

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b) The solution $\hat{\lambda}$ in a) is unique. To show that it is the MLE of λ show that

$$\frac{d^2}{d\lambda^2}\log(L(\lambda))\Big|_{\lambda=\hat{\lambda}} < 0.$$

note! $\sum \log y_i = -n\hat{\lambda}$

$\frac{d^2}{d\lambda^2}\log L(\lambda)\Big|_{\lambda=\hat{\lambda}} = \frac{n}{\lambda^2} + \frac{2\sum \log y_i}{\lambda^3}\Big|_{\lambda=\hat{\lambda}}$ ← ≈6

−5 if 2nd deriv wrong

$= \frac{n}{\hat{\lambda}^2} - \frac{2n\hat{\lambda}}{\hat{\lambda}^3} = \frac{-n}{\hat{\lambda}^2} < 0.$

$= -\frac{n^3}{[\sum \log y_i]^2} < 0$

20

3) Assume that the distribution of x is normal. If the sample size $n = 10$, the sample mean $\overline{x} = 7.05$, and the sample standard deviation $s = 0.4994$, find a 99% confidence interval for μ if possible.

$t_{.005}$	$df = n-1$
3.250	9
99%	

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 7.05 \pm 3.250\ \frac{.4994}{\sqrt{10}}$$

$$= 7.05 \pm .5133 = \boxed{[6.5367,\ 7.5633]}$$

$\overline{20}$

2.576 (6.64, 7.45) −12

4) Suppose two independent random samples were taken yielding $n_1 = 100$, $\overline{Y}_1 = 40.70$, $s_1 = 10.0$. Also $n_2 = 400$, $\overline{Y}_2 = 37.8$, and $s_2 = 40.0$. Find the 90% confidence interval for $\mu_1 - \mu_2$. Hint: do not use the pooled CI.

$$\bar{Y}_1 - \bar{Y}_2 \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

prop's −15

← s_1 s_2 are 10 & 40 −13

$$= (40.70 - 37.8) \pm 1.645\sqrt{\frac{(10)^2}{100} + \frac{(40)^2}{400}}$$

$$= 2.9 \pm 1.645\sqrt{5} = 2.9 \pm 3.678$$

$$= \boxed{[-0.778,\ 6.578]}$$

$\overline{20}$

5) In a local election between Levin and Sanchez, a poll a week before election day indicated that Sanchez would get 42% of the vote (p). What sample size n should be used in an exit poll on election day to estimate p to within a margin of error of 0.04 with 95% confidence? (Do not forget to round up.)

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$$\left(\frac{z_{\alpha/2}}{B}\right)^2 p^*(1-p^*) = \left(\frac{1.96}{.04}\right)^2 (.42)(.58) = 584.88$$

so $n = \boxed{585}$

$\left(\frac{1.96}{.04}\right)^2 \frac{1}{2}\cdot\frac{1}{2} = 601$ →

round up!

$\overline{20}$