Math 483 EXAM 1 covers 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 2.9, 3.1, 3.2, 3.3, and 3.4 and is on Thursday, Feb. 6. You are allowed SIX SHEETS OF NOTES and a CALCULATOR.

A **set** consists of distinct elements enclosed by *braces*, eg $\{1, 5, 7\}$.

The *universal set* S is the set of all elements under consideration.

The *empty set* $\emptyset$ is the set that contains no elements.

A is a subset of B, $A \subseteq B$, if every element in A is in B.

The **union** of A with $B = A \cup B$ is the set of all elements in A or B (or in both).

The **intersection** of A with $B = A \cap B$ is the set of all elements in A and B.

If $A \cap B = \emptyset$, then A and B are (**mutually exclusive** or) **disjoint sets.**

The *complement* of A is $\overline{A}$, the set of elements in S but not in A.

Know DeMorgan's Laws.

The *sample space* S is the set of all possible outcomes of an experiment. A *sample point* E_i is a possible outcome. An *event* is a subset of S. A simple event is a set that contains exactly one element of S, eg $A = \{E_3\}$. A *discrete sample space* consists of a finite or countable number of outcomes.

The *relative frequency interpretation of probability* says that the probability of outcome (sample point) E_i is the proportion of times that E_i would occur if the experiment was repeated again and again infinitely often.

For **any event** A, $0 \leq P(A) \leq 1$.

Three axioms: $P(A) \geq 0, P(S) = 1$, and if $A_1, A_2, ...$ are pairwise mutually exclusive, then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$.

1) **Common problem.** Use order to find S. Using a table to find S if two die are tossed or if a die is tossed twice and to find S if a coin is flipped 2, 3, or 4 times are typical examples. See Q1 6, HW2 A).

The *sample point method* for finding the probability for event A says that if $S = \{E_1, ..., E_k\}$ then $0 \leq P(E_i) \leq 1$, $\sum_{i=1}^{k} P(E_i) = 1$, and $P(A) = \sum_{i:E_i \in A} P(E_i)$. That is, $P(A)$ is the sum of the probabilities of the sample points in A. If all of the outcomes E_i are *equally likely,* then $P(E_i) = 1/k$ and $P(A) =$ (number of outcomes in A)$/k$ if S contains k outcomes.

2) **Common Problem.** Leave the probabilities of some outcomes blank. See Q1 1 and HW2 B.

3) **Common problem.** List all outcomes in S and use these outcomes to find $P(A)$. See HW2 A).

The *multiplication mn rule* says that if there are n_1 ways to do a first task, n_2 ways to do a 2nd task, ..., and n_k ways to do a kth task, then the number of ways to perform the total act of performing the 1st task, then the second task, ..., then the kth task is $n_1 \cdot n_2 \cdot n_3 \cdots n_k$. Techniques for multiplication principle: a) use a slot for each task and write n_i above the ith task. There will be k slots, one for each task. b) use a tree diagram.

A special case is the *number of permutations* (ordered arrangements using r of n distinct objects) $= P_r^n = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1) = \dfrac{n!}{(n-r)!}$. The story problem has r slots and *order is important.* No object is allowed to be repeated in the arrangement. Typical questions include *how many ways to* "choose r people from n and arrange in a line," "to make r letter words with no letter repeated", "to make 7 digit phone numbers with no digit repeated." Key words include *order, no repeated* and *different.*

A special case of permutations is $P_n^n = n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 4 \cdot 3 \cdot 2 \cdot 1 = n \cdot (n-1)! = n \cdot (n-1) \cdot (n-2)! = n \cdot (n-1) \cdot (n-2) \cdot (n-3)! = \cdots$. Typical problems include the number of ways to arrange n books, to arrange the letters in the word CLIPS (5!) etc.

4) **Common Problem:** Use the multiplication mn principle to find *how many* ways to perform the total task. The number of ways to answer k TF questions (2^k) and the number of ways to answer k multiple choice questions with n options (n^k) where $n = 4$ or 5, and the number of serial or license numbers are typical examples. Q1 5, HW3 3, 4, 5, 6

A *combination* is an unordered selection using r of n distinct objects. The *number of combinations* is $C_r^n = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$. This formula is used in story problems where *order is not important.* Key words include *committees, selecting* (eg 4 people from 10), *choose*, *random sample* and *unordered.*

5) **Common problem.** Use the combination formula to solve a story problem. HW3 8.

6) **Common Problem.** Often the multiplication principle will be combined with combinations, permutations, powers and factorials. HW3 7

7) **Common problem.** Use counting rules to find $P(A)$ when the outcomes in S are equally likely. Card problems are typical. See notes.

The *conditional probability* of A *given* B is $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) > 0$. Think of this probability as an experiment with sample space B instead of S. Key word: *given.*

8) **Common Problem.** You are given a table with i rows and j columns and asked to find conditional and unconditional probabilities. Find the row, column, and grand totals. See Q2 1, HW4 C.

Two events A and B are **independent** if any of the following three conditions hold: i) $P(A \cap B) = P(A)P(B)$, ii) $P(A|B) = P(A)$, or iii) $P(B|A) = P(B)$. If *any of these conditions fails to hold,* then A and B are *dependent.*

9) **Common Problem.** Given some of $P(A), P(B), P(A \cap B)$, and $P(A|B)$, find $P(A|B)$ and whether A and B are independent.

10) **Common Problem.** Given a table as in 8), determine if a row event and a column event are independent. Q2 1c, HW4 B).

Multiplication law. $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$. If A and B are ind., then $P(A \cap B) = P(A)P(B)$. If $A_1, A_2, ..., A_k$ are ind., then $P(A_1 \cap A_2 \cap \cdots \cap A_k) = P(A_1) \cdots P(A_k)$. In general $P(A_1 \cap A_2 \cap \cdots \cap A_k) =$
$P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)P(A_4|A_1 \cap A_2 \cap A_3) \cdots P(A_k|A_1 \cap A_2 \bigcap \cdots \cap A_{k-1})$.

Complement rule. $P(A) = 1 - P(\overline{A})$.

Additive law for disjoint events. If A and B are disjoint, then $P(A \cup B) = P(A) + P(B)$. If $A_1, ..., A_k$ are disjoint, then $P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + \cdots + P(A_k)$.

Additive law. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

11) **Common Problem.** Given three of the 4 probabilities above, find the 4th. Variants: given $P(A)$ and $P(B)$ find $P(A \cup B)$ if A and B are disjoint or independent. 11) (continued) Use the addition rule to determine whether A and B are independent or disjoint. See Q2 3, 4, HW4 D), F), G).

A *random variable* (RV) is a real valued function with a sample space as a domain. The *population* is the entire group of objects from which we want information. The *sample* is the part of the pop. actually examined. A RV is *discrete* if it can assume only a finite or countable number of distinct values. The collection of these probabilities is the *probability distribution* of the discrete RV. The *probability function* of a discrete RV Y is $p(y) = P(Y = y)$ where $0 \leq p(y) \leq 1$ and $\sum_{y:p(y)>0} p(y) = 1$.

12) **Common Problem.** The sample space of Y is $S_Y = \{y_1, y_2, ..., y_k\}$ and a table of y_k and $P(y_k)$ is given with one $P(y_k)$ omitted. Find the omitted $p(y_k)$ by using the fact that $\sum_{i=1}^k p(y_i) = p(y_1) + p(y_2) + \cdots + p(y_k) = 1$.

Let Y be a discrete RV with probability function $p(y)$. Then the *mean* or **expected value** of Y is $E(Y) = \mu = \sum_{y:p(y)>0} y\ p(y)$. If $g(Y)$ is a real valued function of Y, then $g(Y)$ is a random variable and $E[g(Y)] = \sum_{y:p(y)>0} g(y)\ p(y)$. The *variance* of Y is $V(Y) = E[(Y - E(Y))^2]$ and the *standard deviation* of Y is SD(Y) $= \sigma = \sqrt{V(Y)}$.

Short cut formula for variance. $V(Y) = E(Y^2) - (E(Y))^2$

If $S_Y = \{y_1, y_2, ..., y_k\}$ then $E(Y) = \sum_{i=1}^{k} y_i p(y_i) = y_1 p(y_1) + y_2 p(y_2) + \cdots + y_k p(y_k)$ and $E[g(Y)] = \sum_{i=1}^{k} g(y_i) p(y_i) = g(y_1)p(y_1) + g(y_2)p(y_2) + \cdots + g(y_k)p(y_k)$. Also $V(Y) = \sum_{i=1}^{k} (y_i - E(Y))^2 p(y_i) = (y_1 - E(Y))^2 p(y_1) + (y_2 - E(Y))^2 p(y_2) + \cdots + (y_k - E(Y))^2 p(y_k)$.
Often using $V(Y) = E(Y^2) - (E(Y))^2$ is simpler where $E(Y^2) = y_1^2 p(y_1) + y_2^2 p(y_2) + \cdots + y_k^2 p(y_k)$.

13) **COMMON PROBLEM.** Given a table of y and $p(y)$, find $E[g(Y)]$ and the standard deviation $\sigma = SD(Y)$. See Q2 5, HW5 C), D).

$E(c) = c$, $E(cg(Y)) = cE(g(Y))$, and $E[\sum_{i=1}^{k} g_i(Y)] = \sum_{i=1}^{k} E[g_i(Y)]$ where c is any constant.

Suppose there are n independent identical trials and Y counts the number of successes and the $p =$ prob of success for any given trial. Let D_i denote a S in the ith trial. Then
i) P(none of the n trials were successes) $= (1-p)^n = P(Y = 0) = P(\overline{D}_1 \cap \overline{D}_2 \cap \cdots \cap \overline{D}_n)$.
ii) P(at least one of the trials was a success) $= 1 - (1-p)^n = P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - P(none) = P(\overline{\overline{D}_1 \cap \overline{D}_2 \cap \cdots \cap \overline{D}_n})$.
iii) P(all n trials were successes) $= p^n = P(Y = n) = P(D_1 \cap D_2 \cap \cdots \cap D_n)$.
iv) P(not all n trials were successes) $= 1 - p^n = P(Y < n) = 1 - P(Y = n) = 1 -$ P(all).
See Q2 2 and HW4 E), H), I).

Know P(Y was at least k) $= P(Y \geq k)$ and P(Y at most k) $= P(Y \leq k)$.

A RV Y is **binomial** if $P(Y = y) = p(y) = \binom{n}{y} p^y q^{n-y}$ for $y = 0, 1, \ldots, n$. Here $q = 1 - p$ and $0 \leq p \leq 1$. $E(Y) = np$, and $V(Y) = np(1-p)$.

14) **Common Problem.** Given a story problem, recognize that Y is bin(n,p), find E(Y), SD(Y), V(Y), P(Y = y), P(Y is at least j) $= p(j) + \cdots + p(n) = 1 - p(0) - \cdots - p(j-1)$, or P(Y is at most j) $= p(0) + \cdots + p(j) = 1 - p(j+1) - p(j+2) - \cdots - p(n)$. Q2 2, HW5 E), F), G), H).