Ch 10] Read §10.1 carefully. 483 72

1]* p 489 The <u>null hypothesis</u> Ho is

a claim about a population parameter such as $H_0: \mu = \mu_0$.

2]* The alternative hypothesis H_A is a competing claim: eg $H_A: \mu \; \begin{matrix} < \\ > \\ \neq \end{matrix} \; \mu_0$.

ex] In ch 10
Ho is of the form "parameter = value"
HA " $\begin{matrix} > \\ < \\ \neq \end{matrix}$ "

Which is correct?
a) $H_0: \mu > 9$ $H_A: \mu = 9$
b) $H_0: \bar{Y} = 9$ $H_A: \bar{Y} > 9$
c) $H_0: \mu = 9$ $H_A: \mu > 9$

3] Typically we would like to have strong evidence that the null hypothesis is false. So the alternative hypothesis is often a research hypothesis.

4] A test statistic is used to decide whether to reject Ho or fail to reject Ho.

5] p491

decision \ truth	Ho	HA
reject Ho	type I error	
fail to reject Ho		type II error

A type I error is made if Ho is

rejected when H_0 is true.
The probability of a type I error
$= \alpha$ = level of test.
A type II error is made

If H_0 is true

if H_0 is not rejected when H_0 is false.
$\beta = P(\text{type II error})$.

6] p 513 The pvalue is the probability, assuming H_0 is true, of obtaining a test statistic at least as extreme or "contradictory" as the test statistic actually observed. Here "contradictory" is determined by the form of H_A.

7] If the pvalue $= 0$, then it was impossible for the test statistic to have occurred if H_0 was true: reject H_0.

If the pvalue $= 1$, then it was impossible for the test statistic to have occurred if H_A was true: fail to reject H_0.

ex] H_0 $\mu = 100$ Sample of size 100
H_A $\mu > 100$ $\sigma = 1$ $\sigma_{\bar{Y}} = \frac{1}{\sqrt{100}} = 0.1$.

If $\bar{Y} = 400$

strong evidence for H_A.

If $\bar{Y} = 90$, H_0 is less contradictory than H_A, although neither seems likely.

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8] Know A test of hypotheses has 4 steps.

i) State Ho and HA.

ii) Calculate the test statistic.

iii) Find the pvalue.

iv) State a conclusion:
If pvalue $\le \alpha$, reject Ho
otherwise fail to reject Ho.
Say in words what rejecting or failing to reject Ho means.

9] * If α is not given, use $\alpha = .05$.

§ 10.3 Some large sample tests

10] * p497 Large Sample test for Ho $\mu = \mu_0$

too strong assumption

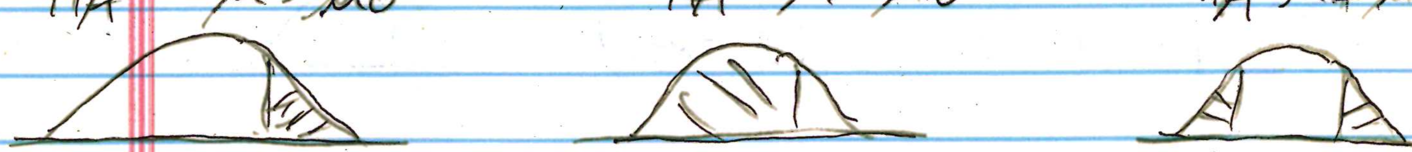
Random sample, $\underline{\sigma \text{ known}}$, CLT holds if Ho is true.

Ho $\mu = \mu_0$ vs HA $\mu \; \substack{> \\ < \\ \neq} \; \mu_0$

test statistic $z_0 = \dfrac{\bar{Y} - \mu_0}{\sigma / \sqrt{n}} \approx N(0,1)$ if Ho is true

Pvalues (Pval = estimated pvalue)

right tail test	left tail test	2 tail test
HA $\mu > \mu_0$	HA $\mu < \mu_0$	HA $\mu \neq \mu_0$



z_0	z_0	$-\lvert z_0 \rvert$ $\lvert z_0 \rvert$
pval $= P(Z > z_0)$	Pval $= P(Z < z_0)$ $= 1 - P(Z > z_0)$	pval $= 2P(Z > \lvert z_0 \rvert)$

where $Z \sim N(0,1)$, and the absolute value $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

$|-1| = 1$, $|2| = 2$, etc.

73.5

11] p511 For a level α 2 tail test, Ho $\mu = \mu_0$ is rejected if μ_0 is outside of a $100(1-\alpha)\%$ CI for μ $\left(= \bar{Y} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}\right)$, otherwise fail to reject Ho. eg a 5% test is rejected if μ_0 is outside of a 95% CI for μ.

(Fail to reject Ho if μ_0 is a reasonable value for μ: ie if $\mu_0 \in 100(1-\alpha)\%$ CI.)

12] To decide the form of HA, choose the form that goes with strong evidence. Tip! differs $\leftrightarrow$ $\neq$

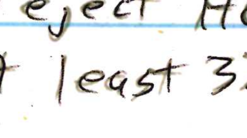
ex] $\bar{Y} = 35$, $\sigma = 5$, $n = 25$

Ford says the Escort gets at least 37 MPG. Test:

i) Ho $\mu = 37$ H_A $\mu > 37$

ii) $z_0 = \dfrac{35-37}{5/\sqrt{25}} = -2$

iii) pval $= P(Z > -2)$



$= 1 - P(Z > 2) = 1 - .0228 = .9772$

iv) pval $> \alpha = .05$ Fail to reject Ho

Escort does not get at least 37 MPG on average.

ex] consumer group says Escort gets less than 37 MPG 483 74

i) $H_0 \ \mu = 37 \qquad H_A \ \mu < 37$

ii) $z_0 = \dfrac{35-37}{5/\sqrt{25}} = -2$

iii) $p\text{val} = P(z < -2) = P(z > 2) = .0228$

iv) $p\text{val} < \alpha = .05$ reject H_0
There is strong evidence that the Escort does not get 37 MPG on average.

ex] $H_0 \ \mu = 100 \qquad \alpha = .05 \qquad z_0 = 2.0$

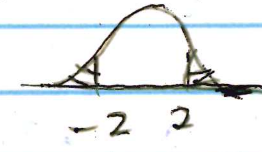
a) right tail $H_A \ \mu > 100$

$p\text{val} = P(z > 2) = .0228 < .05$, reject H_0

b) left tail $H_A \ \mu < 100$

$p\text{val} = P(z < 2) = 1 - P(z > 2) = 1 - .0228 = .9772$
fail to reject H_0

c) 2 tail $H_A \ \mu \neq 100$



$p\text{val} = 2P(z > 2) = 2(.0228) = .0456 < .05$
reject H_0

Go to § 10.8 18]

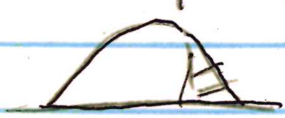
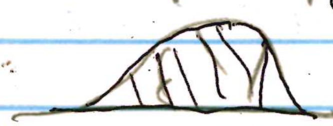
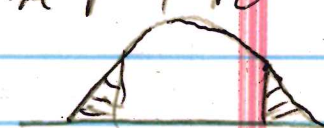
13] p497 Frequently the large sample test statistic is $z_0 = \dfrac{\text{estimate} - \text{parameter}}{\sqrt{\text{estimated variance of estimator}}}$

14] ~~p498~~ know Large sample test for $H_0\ p = p_0$

Random sample, $n > 9 \frac{p_0}{1-p_0}$, $n > 9 \frac{1-p_0}{p_0}$

test statistic $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \approx N(0,1)$ if H_0 is true

pvalues

right tail	left tail	2 tail
$H_A\ p > p_0$	$H_A\ p < p_0$	$H_A: p \neq p_0$
		
z_0	z_0	$-\lvert z_0 \rvert \quad \lvert z_0 \rvert$
$pval = P(Z > z_0)$	$pval = P(Z < z_0) = 1 - P(Z > z_0)$	$pval = 2P(Z > \lvert z_0 \rvert)$

ex] Random sample of 1711 from records of people who died in bicycle crashes from 1987–1991. 386 had blood alcohol levels above .1%. Test if $p < 0.25$: if $\alpha = .01$. $\hat{p} = \frac{386}{1711} = .2256$

i) $H_0\ p = .25 \qquad H_A\ p < .25$

ii) $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.2256 - .25}{\sqrt{\frac{.25(1-.25)}{1711}}} = \frac{-0.0244}{.0105}$

α	.005	.01 left tail
z	-2.576	-2.326

$= -2.33$ $\qquad .005 < pval < .01$

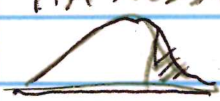
iii) $pval = P(Z < -2.33) = P(Z > 2.33) = .0099$

iv) $pval = .0099 < .01$ so reject H_0, the proportion of fatally injured cyclists that were drunk < 0.25.

15] Know Large sample test for H_0 $\mu = \mu_0$, σ unknown
random sample, $n \geq 30$, CLT holds if H_0 is true (actually the t test)

test statistic $z_0 = \dfrac{\bar{Y} - \mu_0}{s/\sqrt{n}} \approx N(0,1)$ if H_0 is true

pvals

right tail H_A $\mu > \mu_0$	left tail H_A $\mu < \mu_0$	2 tail H_A $\mu \neq \mu_0$
		
z_0	z_0	$-\lvert z_0\rvert$ $\lvert z_0\rvert$
$P(Z > z_0)$	$P(Z < z_0) = 1 - P(Z > z_0)$	$2P(Z > \lvert z_0\rvert)$

ex) $n = 32$ $z_0 = 1.97$ H_A $\mu > 0$
4 step test

i) H_0 $\mu = 0$ H_A $\mu > 0$
ii) $z_0 = 1.97$ $.01 < pval < .025$ z 1.96 2.326
iii) $P(Z > 1.97) = .0244$ .025 .01 right tail
iv) pval $< \alpha = .05$ reject H_0

16] Large sample test for
H_0 $\mu_1 - \mu_2 = D_0$ 2 ind random samples $n_1 > 30$, $n_2 > 30$ CLT holds for $\bar{Y}_1$ $\bar{Y}_2$

test statistic $z_0 = \dfrac{(\bar{Y}_1 - \bar{Y}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \approx N(0,1)$ if H_0 is true

pvalues same as always
usually $D_0 = 0$
4 step test is the same

ex] Bank has 2 proposals to increase the amount of credit charged on its credit cards. Each proposal is offered to a random sample of 150 bank customers.

sometimes s_i^2 is given

Proposal	n_i	$\bar{Y}_i$	s_i
1	150	1987	392
2	150	2056	413

Test whether μ_1 and μ_2 are <u>different</u>.

soln i) $H_0\ \mu_1 - \mu_2 = 0 \qquad H_A\ \mu_1 - \mu_2 \neq 0$

ii) $$z_0 = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1987 - 2056}{\sqrt{\frac{(392)^2}{150} + \frac{(413)^2}{150}}} = -1.48$$

iii) pval $= 2P(z > 1.48) = 2(.0694) = .1388$

$.1 < \text{pval} < 1$

iv) pval $> .05$, fail to reject H_0
The mean charge amounts from the 2 proposals are the same.

z	−1.645	0
	.1	1

two tail

17) **Know** large sample test for Ho $p_1 - p_2 = 0$

n_1 $\quad y_1$ or $\hat{p}_1 = \frac{y_1}{n_1}$
n_2 $\quad \underbrace{y_2}_{\#\text{ of successes}}$ or $\hat{p}_2 = \frac{y_2}{n_2}$ $\quad$ > 2 independent random samples

$n_i > 9 \dfrac{\hat{p}_i}{1-\hat{p}_i}$, $n_i > 9 \dfrac{1-\hat{p}_i}{\hat{p}_i}$, $i = 1, 2$

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad \text{where } \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

ex]

		# who say women are safer drivers than men	
100	men	31	$\hat{p}_1 = .31$
100	women	67	$\hat{p}_2 = .67$

Test if the proportion of men who say women are safer is smaller than the prop of women who say women are safer

i) Ho $\underset{p_1 = p_2}{p_1 - p_2 = 0}$ $\qquad$ HA $\underset{p_1 < p_2}{p_1 - p_2 < 0}$

ii) $$\hat{p} = \frac{100(.31) + 100(.67)}{100 + 100} = 0.49$$

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.31 - .67}{\sqrt{.49(.51)\left(\frac{1}{100} + \frac{1}{100}\right)}}$$

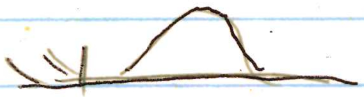
$$= \frac{-0.36}{\sqrt{.00499}} = -5.09$$

over for p-value with z table

left tail

$Z \quad -2.57$

or pval < .005

iii) 

pval = 0.0

iv) reject Ho, the proportion of men who say that women are safer drivers than men is smaller than the proportion of women who say so.

§ 10.8

p521

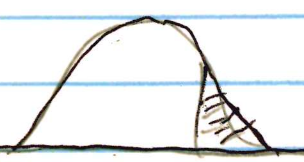
18) Know t test for Ho $\mu = \mu_0$

Random sample $n \geq 2$ CLT holds $df = n-1$

$$t_0 = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} \approx \begin{cases} t & \text{if } df \leq 29 \\ N(0,1) & df > 29 \end{cases}$$

use t table if $df \leq 29$
Z table if $df > 29$

right tail
HA $\mu > \mu_0$



$pval = P(T_{n-1} \geq t_0)$

left tail
HA $\mu < \mu_0$

$pval = P(T_{n-1} \leq t_0)$
$= 1 - P(T_{n-1} > t_0)$

2 tail
HA $\mu \neq \mu_0$

$pval = 2\, P(T_{n-1} > |t_0|)$

where T_{n-1} is a t dist with $df = n-1$

same 4 step procedure i) state Ho and HA
ii) calculate test statistic t_0 iii) find pval iv) give conclusion in nontechnical terms

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ex] pvalues for t test

random sample from normal pop

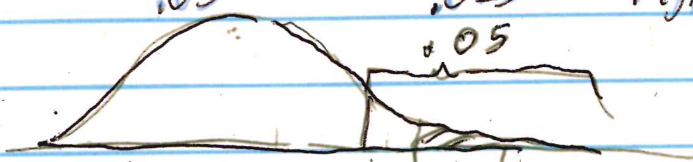
Ho $\mu = 0$ $\quad n = 15$

a) pval if H_A $\mu > 0$ $\quad t_0 = 1.97$

Find the values from the t table that bracket t_0

$t_{.05}$	$t_{.025}$	df
1.761	2.145	14
.05	.025	right tail

.05



1.761 1.97 2.145

.025

so $.025 < \text{pval} < .05$

(reject Ho if $\alpha = .05$
fail to reject Ho if $\alpha = .01$)

b) H_A $\mu > 0$ $\quad t_0$ -1.97

-1.97

-2.145	-1.761	df
.975	.95	14 right tail

pval > 0.5 $\quad .95 < \text{pval} < .975$

fail to reject Ho

in fact $1 - .05 = .95 < \text{pval} < .975 = 1 - .025$

c) H_A $\mu < 0$ $\quad t_0 = 1.97$

0 1.97

1.761	2.145	df
.95	.975	14 left tail

pval > 0.5 $\quad .95 < \text{pval} < .975$

fail to reject Ho

d) $H_A \quad \mu \neq 0 \qquad t_0 = 1.97$ $\quad$ 1.761 $\quad$ 2.145

.1 ⟶ .1 $\quad$ .05

$2(.025) = .05 < pval < .1 = 2(.05)$

so fail to reject Ho

e) $H_A \quad \mu > 0 \qquad t_0 = 1.97 \quad n = 32$

use z table

$pval = P(z > 1.97) = .0244$

good check ⟶

1.96	2.326	z
.025	.01	

$.01 < pval < .025$

df

right tail

ex) $\alpha = .01$ Golfer tries out club to see if new club drives balls further than old club. With old club she believes she can hit the ball 200 yards. She will buy the new club if it is better. Shop lets her take 10 drives. $n = 10, \; \bar{y} = 204.60 \qquad s = 9.03$

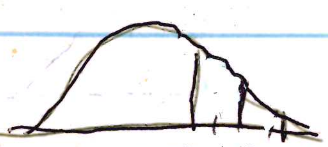
Soln] μ = mean drive length with new club

i) $H_0 \quad \mu = 200 \qquad H_A \quad \mu > 200$ (ho is never from the sample)

ii) $t_0 = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{204.6 - 200}{9.03/\sqrt{10}} = 1.611$

iii) $df = n-1 = 9$

$t_{.10}$	$t_{.05}$	df
1.383	1.833	9



$.05 < pval < .1$

iv) fail to reject Ho $\quad$ not enough evidence to conclude that new club is better than old.