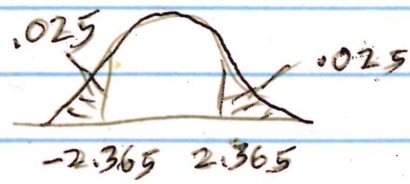
ex] Test whether the mean amount of relief from a sugar pill differs from 0 if $n=8$, $\bar{Y}=4$, $t_0 = 2.365$ and $\alpha = .03$

i) $H_0\ \mu = 0 \qquad H_A\ \mu \neq 0$

ii) $t_0 = 2.365$ (given)

iii) $df = n-1 = 7$

$t_{.025}$	df
2.365	7
.05	twotail



$$\underline{pval} = 2(.025) = \underline{.05}$$

iv) $pval = .05 > \alpha = .03$ fail to reject H_0

Tendinzed mean amount of relief is 0.

19) tips for using t table for t-test

i) pval from Z table is often close, especially if $df > 7$.

Now suppose $1.8 < t_0 < 2.1$, $t_0 = 1.96$

$t_{.05}$	$t_{.025}$	$df = n-1$
1.761	2.145	14
		n-1

$\uparrow$ t_0 $|t_0|$

ii) for right tail test $.025 < pval < .05$

iii) for 2 tail test $2(.025) = .05 < pval < .1 = 2(.05)$

iv) for left tail test $1-.05 = .95 < pval < .975 = 1-.025$

v) For general t_0 sketch a picture. For left and right tail tests, you can immediately tell whether pval > 0.5 or pval < 0.5.

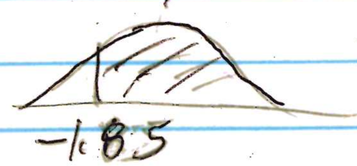
vi) If df > 29, use Z table to get pvalue.

vii) If $-2.1 < t_0 < -1.8$, use symmetry

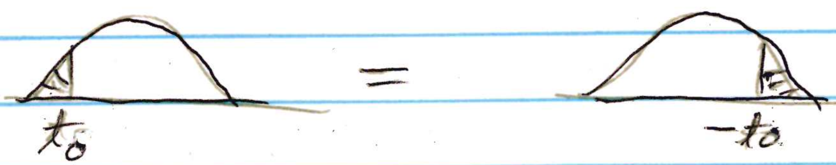
$t_{.05}$	$t_{.025}$	$df = n-1$
1.761	2.145	14

$\uparrow$
$-t_0$
$|t_0|$

a) right tail → pval > 0.5
or $1 - .05 = .95 < \text{pval} < .975 = 1 - .025$



b) left tail $.025 < \text{pval} < .05$ by symmetry



c) 2 tail $2(.025) = .05 < \text{pval} < .1 = 2(.05)$

ex] Researchers measured the amount of D-gluclose in cockroach hindguts.

5 measurements $\bar{Y} = 44.44$, $s = 20.741$.

Test whether the mean amount of D-gluclose is greater than 54.44.

i) H_0 $\mu = 54.44$ H_A $\mu > 54.44$

ii) $t_0 = \dfrac{\bar{Y} - \mu_0}{s/\sqrt{n}} = \dfrac{44.44 - 54.44}{20.741/\sqrt{5}} = \dfrac{-10}{9.276} = -1.078$

iii)

		$df = n-1$
-2.132	0	4
.95	.5	right tail

$0.5 < pval < 0.95$

iv) fail to reject H_0 The mean amount of D gluclose is 54.44 (or not greater than 54.244). go to 13)

end 2005

Not on exams

(bad test if normality fails)

§10.9 1] $Y_1, \dots, Y_n$ iid $N(\mu, \sigma^2)$

p531 H_0 $\sigma^2 = \sigma_0^2$

test statistic $X_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2} \overset{H_0}{\sim} \chi^2_{n-1}$ $df = n-1$

Pvalues

right tail

H_A $\sigma^2 > \sigma_0^2$

$P(\chi^2_{n-1} > X_0^2)$

left tail

H_A $\sigma^2 < \sigma_0^2$

$P(\chi^2_{n-1} < X_0^2)$

2 tail

H_A $\sigma^2 \neq \sigma_0^2$

reject H_0 if $X_0^2 < \chi^2_{\alpha/2}$ or $X_0^2 > \chi^2_{1-\frac{\alpha}{2}}$

$\S 10.10$ not on exams

79.5

2] Power = P(reject H_0 when parameter $=\theta$)

= P(test statistic is in rejection region $|\theta$)

ex] $Z_0 \overset{H_0}{\approx} N(0,1)$ $\quad \alpha = .05$

H_A $\mu > \mu_0$ reject H_0 if $Z_0 > 1.645$

H_A $\mu < \mu_0$ reject H_0 if $Z_0 < -1.645$

H_A $\mu \neq \mu_0$ reject H_0 if $Z_0 > 1.96$ or $Z_0 < -1.96$

power $=\alpha$ if H_0 is true.

suppose $n=100$ $\quad t_0 = \dfrac{\bar{X}-\mu_0}{S/\sqrt{n}} \overset{H_0\ \mu=100}{\approx} N(0,1)$ $\quad S \approx 10 \approx \sigma$

H_A $\mu > 100$

Power for $\mu = 100$ $= P(t_0 > 1.645) \approx .05$

Power for $\mu = 110$ $\quad \dfrac{\bar{X}-110}{S/\sqrt{n}} \approx N(0,1)$, $X \sim N(110, 10^2)$

$\bar{X} \approx N(110, 1)$, $\frac{S}{\sqrt{n}} \approx 1$

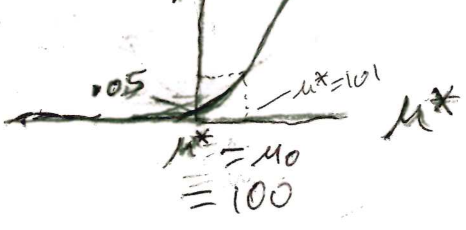
So $t_0 = \dfrac{\bar{X}-100}{S/\sqrt{n}} \approx N(10, 1)$

Power $\approx P(N(10,1) > 1.645) = P(Z > 1.645 - 10) \approx 1.00$

Power for μ^* $\quad t_0 = \dfrac{\bar{X}-100}{S/\sqrt{n}} \approx N(\mu^*-100, 1)$

power $\approx P(Z > 1.645 - (\mu^* - 100))$

ex] $\mu^* = 101$, Power $\approx P(Z > 0.645) = \dfrac{.2611 + .2578}{2} = .25945$



3] not on exams

p542 Neyman Pearson test of

$H_0\ \theta = \theta_0$ vs $H_A\ \theta = \theta_a$.

has rejection region

reject H_0 if $\frac{L(\theta_0)}{L(\theta_a)} < K$ where

$P_{\theta_0}\left(\frac{L(\theta_0)}{L(\theta_a)} < K\right) = \alpha \doteq .05.$ (if L is likelihood of continuous RV)

ex] $Y_1, \ldots, Y_n \sim N(\mu, 1)$

$H_0\ \mu = 0$ $H_A\ \mu = 1$

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y_i - \mu)^2} = \left(\frac{1}{\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\Sigma(y_i-\mu)^2}.$$

$$\text{So } \frac{L(0)}{L(1)} = \frac{e^{-\frac{1}{2}\Sigma y_i^2}}{e^{-\frac{1}{2}\Sigma(y_i-1)^2}} = e^{-\frac{1}{2}\left[\Sigma y_i^2 - \Sigma(y_i-1)^2\right]}$$

$$= e^{-\frac{1}{2}\left[\Sigma y_i^2 - (\Sigma y_i^2 - 2\Sigma y_i + n)\right]}$$

$$= e^{-\frac{1}{2}2\Sigma y_i - \frac{n}{2}}$$

reject H_0 if $-\frac{1}{2}\Sigma y_i - \frac{n}{2} < K'$

or if $-\frac{1}{2}\Sigma y_i < k_1$

or $\Sigma y_i > k_2$

or $\bar{Y} > c$

$\bar{Y} \overset{H_0}{\sim} N(0, \frac{\sigma^2}{n})$

$Z = \frac{\bar{Y} - 0}{\sigma/\sqrt{n}}$, $\sigma^2 = 1$

where $\alpha = P_0(\bar{Y} > c) = .05 = P\left(\frac{\bar{Y}-0}{1/\sqrt{n}} > \frac{c-0}{1/\sqrt{n}}\right) = P(Z > \sqrt{n}c)$

or $\sqrt{n}c = 1.645$ so $c = \frac{1.645}{\sqrt{n}}$.

$\bar{Y} \overset{H_1}{\sim} N(1, \frac{1}{\sqrt{n}})$. power for $\mu = 1$ $= P_1\left(\bar{Y} > \frac{1.645}{\sqrt{n}}\right) = P\left(\frac{\bar{Y}-1}{1/\sqrt{n}} > \frac{\frac{1.645}{\sqrt{n}} - 1}{1/\sqrt{n}}\right)$

$= P(Z > 1.645 - \sqrt{n}) \overset{n=4}{=} P(Z > -.355) = 1 - \left(\frac{.3669 + .3632}{2}\right) = .63495$

4) not on exams

p550 Likelihood ratio test has test statistic $\lambda(\underset{\sim}{y}) = \dfrac{L(\hat{\Theta}_0)}{L(\hat{\Theta})}$ where (80.5

$\underbrace{\hat{\Theta} \text{ is the MLE}}_{\text{parameter space} = \Theta}$ and $\hat{\Theta}_0$ is $\underbrace{\text{the MLE under } H_0.}_{\text{parameter space } \Theta_0 \subseteq \Theta}$

Reject H_0 if $\lambda(\underset{\sim}{y}) < k$ where $\alpha = \sup_{\theta \in \Theta_0} P_\theta(\lambda(\underset{\sim}{y}) < k)$.

Do 3 or so old Quiz 11's