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Predict Y from X

Math 484 Quiz 4 fall 2016 Name___________________

1) Suppose a scientist desires to predict head breadth from cranial capacity. Assume that the correlation $r = 0.6665$. $= \hat{\rho}_1$

$\hat{\beta}_2 = 10.63$
$\hat{\beta}_1 = -171.46$

−9

	variable	mean	standard deviation
Y	head breadth	140.79	7.4976
X	cranial capacity	1325.9	119.64

a) Find the slope of the least square line.

$$\hat{\beta}_2 = \hat{\rho}\frac{s_Y}{s_X} = 0.6665 \frac{7.4976}{119.64} = \boxed{0.04177}$$

b) Find the intercept of the least square line.

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 140.79 - 0.04177(1325.9)$$
$$= \boxed{85.4095}$$

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Label      Estimate      Std. Error   t-value    p-value
Constant   -0.0853881    0.184315      -0.463     0.6467
D           0.151516     0.00563864    26.871     0.0000
Ht          0.0144717    0.00277705     5.211     0.0000
(D Ht) = (13 76), Prediction = 2.98417, s(pred) = 0.0841639, s = 0.0149419
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2) The output above is from a data set that had $n = 31$ cases. Cherry trees have valuable wood. Suppose that we desire to predict the cube root of the volume of the tree $Y = Vol^{1/3}$ from the tree trunk diameter D and the tree height Ht. If $D = 13$ and $Ht = 76$, so that $\boldsymbol{x}_f = (1, 13, 76)$, assume that $\hat{Y}_f = 2.98417$, $se(\hat{Y}_f) = 0.0149419$ and that se(pred) $= 0.0841639$.

$df = n - p = 31 - 3 = 28$ | 2.048 95%

a) If $\boldsymbol{x}_f = (1, 13, 76)^T$ find a 95% confidence interval for $E(Y_f|\boldsymbol{x}_f)$.

$$\hat{Y} \pm t_{n-p,1-\frac{\alpha}{2}} SE(\hat{Y}_f) = 2.98417 \pm 2.048(0.0149419)$$
$$= \boxed{[2.9530, 3.0148]}$$

b) If $\boldsymbol{x}_f = (1, 13, 76)^T$ find a 95% prediction interval for Y_f.

$$\hat{Y} \pm t_{n-p,1-\frac{\alpha}{2}} SE(pred) =$$
$$2.98417 \pm 2.048(0.0841693)$$

$$= \boxed{[2.8118, 3.1565]}$$

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x_i	y_i	$x_i - \overline{x}$	$y_i - \overline{y}$	$(x_i - \overline{x})(y_i - \overline{y})$	$(x_i - \overline{x})^2$
7	128	−5.4	−94.4	509.76	29.16
12	213	−0.4	−9.4	3.76	0.16
4	75	−8.4	−147.4	1238.16	70.56
14	250	1.6	27.6	44.16	2.56
25	446	12.6	223.6	2817.36	158.76
sum 62	1112			4613.2	261.2

+2

sum 62 1112 4613.2 $261.2 = \Sigma(X_i - \bar{X})^2$

mean $\frac{62}{5} = \bar{X} = 12.4$ $1112/5 = \bar{Y} = 222.4$ $= \Sigma(X_i - \bar{X})(Y_i - \bar{Y})$

3) In the above table, x_i is the number of galleys of type set and y_i is the dollar cost of correcting typographical errors in a random sample of a firm specializing in technical printing around 1980.

a) Complete the table and find the least squares estimators $\hat{\beta}_1$ and $\hat{\beta}_2$. (In the table, use 4 or 5 decimal places, then round.)

or −10 then prop

$$\hat{\beta}_2 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} = \frac{4613.2}{261.2} = \boxed{17.6616}$$

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$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 222.4 - 17.6616(12.4) = \boxed{3.3966}$$

~3.40

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b) Predict y if $x = 13$.

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X = 3.3966 + 17.6616(13)$$

$$= \boxed{232.9974}$$

~233.0

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