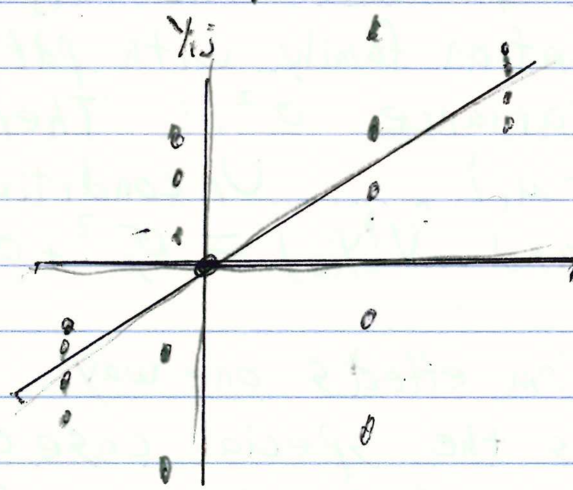
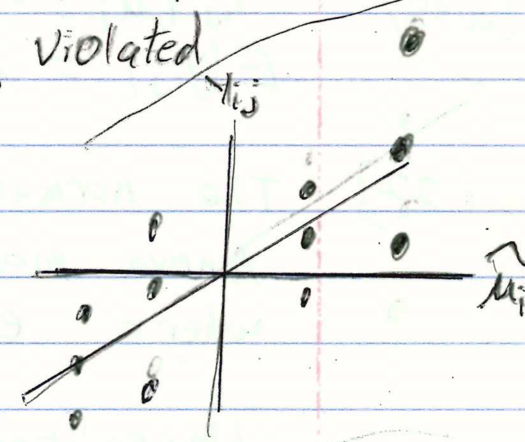
19] Know p183 Let S_i be the sample standard deviation of the ith group. If $\max(S_1, \ldots, S_p) \le 2 \min(S_1, \ldots, S_p)$,

plots: use range of dot plot

then the oneway Anova F test pval is approx correct if the response and residual plots suggest that the remaining one way Anova assumptions are reasonable.



← constant variance assumption is violated



point 19] may be satisfied

20] * p189 For the random effects one way Anova model, the p levels of the factor are a random sample of p levels from some pop of levels Λ_F.

For a fixed effects oneway Anova model, the p factor levels are fixed.

21] *p189 The cell means model for the random effects one way Anova model is $Y_{ij} = \mu_i + e_{ij}$ for $i = 1, \ldots, p$

and $j = 1, \ldots, n_i$. The μ_i are randomly selected from some pop Λ with mean $E(\mu_i) = \mu$ and $V(\mu_i) = \sigma_\mu^2$, where $i \in \Lambda_F$

is equivalent to $\mu_i \in \Lambda$. The e_{ij} and μ_i are independent. The e_{ij} are iid from a location family with pdf f, mean 0 and variance σ^2. Then $Y_{ij} | \mu_i \sim f(y - \mu_i)$. Unconditionally $E(Y_{ij}) = \mu$ and $V(Y_{ij}) = \sigma_\mu^2 + \sigma^2$.

for inference to be correct

22] The normal random effects one way Anova model is the special case of 21] where $e_{ij} \sim N(0, \sigma^2)$ and $\mu_i \sim N(\mu, \sigma_\mu^2)$.

Unconditionally $Y_{ij} \sim N(\mu, \sigma_\mu^2 + \sigma^2)$.

23] Know for E3, Final random effects one way Anova F test

i) $H_0 \;\; \sigma_\mu^2 = 0 \quad H_A \;\; \sigma_\mu^2 > 0$

ii) $F_0 = \dfrac{MSTR}{MSE}$

iii) $pval = P(F_{p-1, n-p} > F_0)$

$\Bigg\}$ usually from output

iv) If $pval < \delta$ reject H_0, conclude $\sigma_\mu^2 > 0$ and the mean response depends on the factor level.

If pval $\geq \delta$ fail to reject Ho,
conclude $\sigma_\mu^2 = 0$ and the mean response
does not depend on the factor level.

See ex 5.8

Kuehl p129-134

ex] A metal alloy is produced by a high temperature casting procedure. Three randomly selected fabrications at the same factory were selected. A random sample of 10 bars was selected from each fabrication, and the tensile strength of each bar was measured.

Clue: random twice

Anova	df	SS	MS	F	P
trt	2	147.88	73.94	12.708	0.0001
error	27	157.10	5.82		

i) Is this a fixed or random effects one way Anova?
ii) Do the 4 step test.

soln] i) random effects: taking a random sample of fabrications from pop of all possible fabrications

ii) Ho $\sigma_\mu^2 = 0$ $\quad H_A$ $\sigma_\mu^2 > 0$

$F_0 = 12.708$

pval $= .001$

reject Ho, $\sigma_\mu^2 > 0$. The mean tensile strength depends on fabrication.

(5.2.5)

24) * p191 For DOE a transformation plot is a plot of $\widehat{t_\lambda(Z)}$ vs $t_\lambda(Z)$

but $\Lambda_L = \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$

$$t_\lambda(Z) = \begin{cases} Z^\lambda & \lambda \neq 0 \\ \log Z & \lambda = 0 \end{cases}$$

The best transformation plot corresponds to the transformation to use.

For one way Anova models with $n_i \equiv m \geq 5$, look for a transformation plot where

for inference to be correct

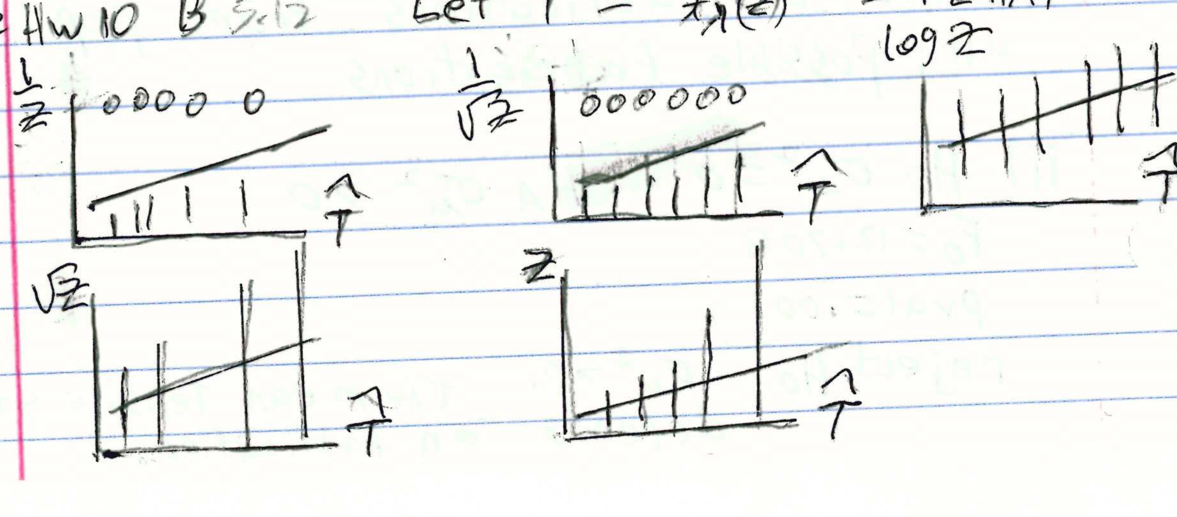
i) the p dot plots scatter about the identity line with roughly similar shape and spread.

ii) dot plots with more skew are worse than dot plots with less skew or which are $\approx$ symmetric

iii) spread that decreases or increases with TZHAT is bad.

25) E3, Final Prob: p191-2 Choose a transformation from transformation plots see Fig 5.4 and ex 5.4 on p 192

ex) like HW 10 B 5.12 Let $\hat{T} = \widehat{t_\lambda(Z)}$ = TZHAT



Q] Which transformation is best? Explain briefly.

A] $Y = \log(Z)$ since it has the best transformation plot.

$\frac{1}{Z}$ and $\frac{1}{\sqrt{Z}}$ have skewed dot plots and the dot plots don't cover the identity line. $\sqrt{Z}$ and Z have spread that increases with $\hat{T}$.

26] For $Y = t_{\lambda^*}(Z)$ ← best, the transformation plot is the response plot. Make the usual checks on the model, including the residual plot.

27] For a plot of $\hat{Z}$ vs Z, the log rule and ladder rule for Z are useful: eg $\frac{\max Z_{ij}}{\min Z_{ij}} > 10 \rightarrow$ use $Y_{ij} = \log Z_{ij}$

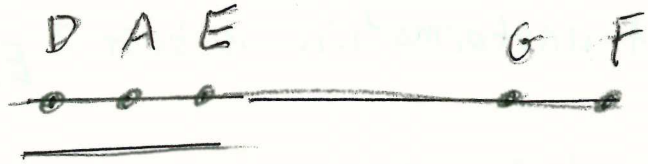
28] If H_0 $\mu_1 = \cdots = \mu_p$ is rejected, want to know which means significantly differ and which do not significantly differ. Two methods i) contrasts and multiple comparisons ii) graphical Anova.

response plot is useful

29) Multiple comparisons place ordered means in a line and uses symbols or lines for groups of nonsignificant means.

assumes $\sigma_i^2 \equiv \sigma^2$

ex]



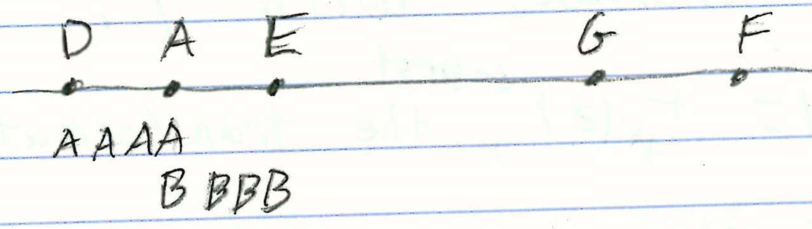
ordered sample means

line →

conclude $\mu_A = \mu_D$, $\mu_A = \mu_E$ and $\mu_D = \mu_E$.
So A, D and E are a group of (pop) means that are not significantly different.

(depends on n)

ex]



symbols

statistically significant

conclude $\mu_A = \mu_D$ and $\mu_A = \mu_E$ but $\mu_D \neq \mu_E$

short for not statistically significant

(there is enough evidence to conclude $\mu_D \neq \mu_E$, but not enough evidence to conclude $\mu_A \neq \mu_E$).

Groups of means that are not significantly different are A and D and A and E.

30] For E3, Final, be able to find groups of means that are not significantly different, as in the above ex's.

See ex 5.6.

31] Graphical Anova uses the residuals as a reference set, instead of a t, F or normal distribution.

still need $\sigma_i^2 = \sigma^2$

Let the scaled treatment deviations of effects be $\sqrt{\frac{n-p}{p-1}}\,(\overline{Y}_{i0} - \overline{Y}_{00})$

$= \sqrt{\frac{n-p}{p-1}}\,(\hat{\mu}_i - \overline{Y}_{00})$. A dot plot

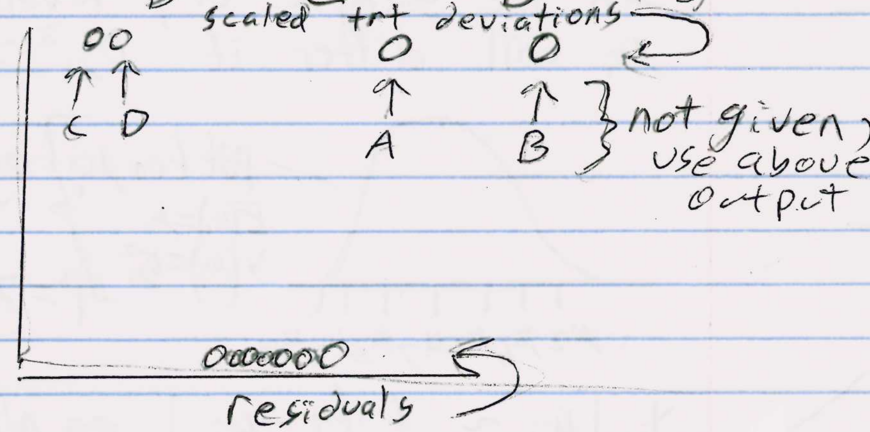
of the scaled deviations is placed above a dot plot of the residuals. For $n \le 40$, declare pop means μ_A and μ_B significantly different if the distance between their scaled deviations is more than the range of the residuals. Otherwise μ_A and μ_B are not statistically significant.

ex 5.7 R output)

Scaled dev or smn .029 .066 −.051 −.045 } output

Trt A B C D

Plot



output has $\overline{x}_C < \overline{x}_D < \overline{x}_A < \overline{x}_B$ from scaled deviations

C and D are not statistically significant A and B means are significantly different from C and D means. Marginal evidence that A and B means are significantly different.

32) Both the fixed and random effects 1way Anova tests are testing whether

an overall mean μ can be used (545)

or if the mean depends on the treatment (level).

reject H_0: use $\hat{\mu}_i = \overline{Y}_{i0} = \hat{Y}_{ij}$

fail to reject H_0: use $\hat{\mu} = \overline{Y}_{00}$.

Analogy: MLR reject H_0: use $\hat{Y}_i = \underset{\sim}{x}_i^T \hat{\underset{\sim}{\beta}}$

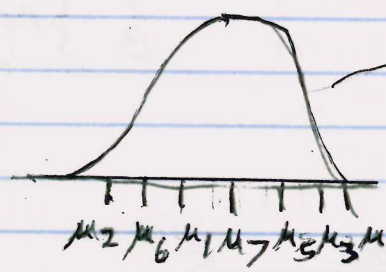
fail to reject H_0: use $\hat{Y}_i = \overline{Y}$

33) * With a random effects 1 way Anova model, the conclusions can be generalized to the entire pop of levels.

For the fixed effects 1 way Anova model, the conclusions only hold for the p levels.

34) For random effects, $E(\mu_i) = \mu$, $V(\mu_i) = \sigma_\mu^2$.

In the sample of p levels, the μ_i will differ if $\sigma_\mu^2 > 0$.



pdf for μ_i eg $N(\mu, \sigma_\mu^2)$

$E(\mu_i) = \mu$

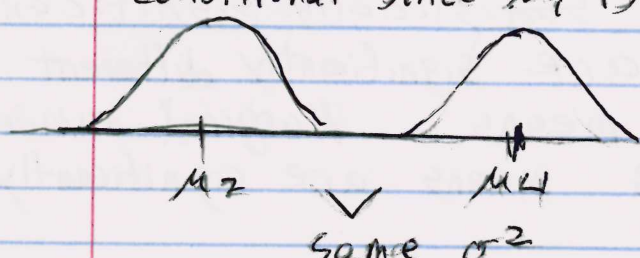
$V(\mu_i) = \sigma_\mu^2$

$p = 7$ random samples

for inference to work

$\underbrace{Y_{ij} \mid \mu_i}$ $\sim f(y - \mu_i)$, eg $N(\mu_i, \sigma^2)$, $j = 1, \ldots, n_i$

conditional since μ_i is random (constant given μ_i)



same σ^2

35) Fixed effects 1 way Anova F test: tested $H_0\ \mu_1 = \cdots = \mu_p$

random effects 1 way Anova F test: want to know if $\mu_i = \mu$ for every level i in the pop of levels Λ (Λ not necessarily finite).

Q] uestion Since $E(\mu_i) = \mu$ and $V(\mu_i) = \sigma_\mu^2$, when is $\mu_i \equiv \mu \quad \forall\, i \in \Lambda$?

A] nswer If $\sigma_\mu^2 = 0$.

36] Back to fixed effects: multiple comparisons when $H_0\ \mu_1 = \cdots = \mu_p$ is rejected.

p185 A contrast $C = \sum_{i=1}^{p} k_i \mu_i$ where $\sum_{i=1}^{p} k_i = 0$.

The estimated contrast $\hat{C} = \sum_{i=1}^{p} k_i \hat{\mu}_i$.

need $\sigma_i^2 \equiv \sigma^2$

$$SE(\hat{C}) = \sqrt{MSE \sum_{i=1}^{p} \frac{k_i^2}{n_i}}.$$

37] Contrasts should be picked before doing the experiment to avoid data snooping.

ex] $C_{ij} = \mu_i - \mu_j$ compares μ_i with μ_j

$C = \mu_1 - \dfrac{\mu_2 + \cdots + \mu_p}{p-1}$ compares the last $p-1$ groups (trts) with the 1st group (= often control or standard).

38] There are exceptions to 37].
Consider the family of null hypotheses for contrasts $\{H_0: \sum_{i=1}^{p} k_i \mu_i = 0$ where $\sum_{i=1}^{p} k_i = 0$ and the k_i may satisfy other constraints$\}$.

Let δ_S = prob of a type I error (rejecting H_0 when H_0 is true) for a single test from this family. 55.5

Let $\delta_T = P$(of making at least one type I error among the family of contrasts). Let the family level δ_F be an upper bound on (the usually unknown size) δ_T.

Often take $\delta_F = 0.05$. Then

$1-\delta_T = P$(no type I errors among the family).

$1-\delta_F = .95 \le 1-\delta_T$.

39] Two important families of contrasts are the family of all possible contrasts and the family of pairwise differences $c_{ij} = \mu_i - \mu_j$ for $i \neq j$.

Scheffé multiple comparisons has a δ_F for the family of all possible contrasts while the Tukey multiple comparisons procedure has a δ_F for the family of all $\binom{p}{2}$ pairwise contrasts.

40] If you find M (≥ 10) contrasts of interest after looking at the data, use the Scheffe procedure.
Use the Tukey procedure for pairwise contrasts.

If $\delta_F = .05$, $.95 \le P$(no type I errors among all $\binom{p}{2}$ pairwise contrasts using the Tukey procedure
or among the M contrasts looked at using Scheffe's method), where if you perform 1000 experiments, about 950 will have no type I errors among the family of tests.