Math 501 HW 1 Spring 2025. Due Friday, Jan. 24.

Place your solutions on a separate sheet of paper. DO NOT place solutions side by side. You may use both the front and the back of each sheet.

YOU ARE BEING GRADED FOR WORK NOT JUST THE FINAL ANSWER. As a rule of thumb, you should have some idea of what you were doing, even without the book or notes. You are encouraged to form groups to discuss ideas and HW problems, but do not copy.

Exam 1 review may be useful. For the quiz, the exam 1 review and qual problems from the course website may be useful. 3 sheets of notes for the quiz.

One way to show that $A = B$ is to show i) if $x \in A$ then $x \in B$ so $A \subseteq B$, and ii) if $x \in B$ then $x \in A$ so $B \subseteq A$.

(R. #9, p. 16) is short for (Royden, problem #9 on page 16).

Note that iff = if and only if = $\Leftrightarrow$ = "is equivalent to." To show LHS iff RHS, show LHS $\Rightarrow$ RHS and show RHS $\Rightarrow$ LHS.
(e.g. LHS = left hand side and RHS = right hand side)

1) (R. #9, p. 16): Show $A \subseteq B$ iff $A \cap B = A$ iff $A \cup B = B$.

Hint: a) Show $A \subseteq B \Rightarrow A \cap B = A$. b) Show $A \cap B = A \Rightarrow A \subseteq B$. c) Show $A \subseteq B \Rightarrow A \cup B = B$. d) Show $A \cup B = B \Rightarrow A \subseteq B$.

2) (R. #17, p. 16): Let $f : X \to Y$.
a) Show $f^{-1}[\cup_{\lambda \in \Lambda} B_\lambda] = \cup_{\lambda \in \Lambda} f^{-1}[B_\lambda]$.
b) Show $f^{-1}[\cap_{\lambda \in \Lambda} B_\lambda] = \cap_{\lambda \in \Lambda} f^{-1}[B_\lambda]$.
c) Show $f^{-1}[B^c] = [f^{-1}[B]]^c$ for $B \subseteq Y$.

(Note: by the definition of $f^{-1}[A]$, it follows that $A \subseteq Y$. Technically $B \subseteq Y$ and $B^c = Y - B = \{y \in Y : y \notin B\}$.)

3) (Similar to R. #12a) on p. 16.): Show that $A \triangle B = B \triangle A$.

4) Let $X = \{1, 2, 3, 4\}$ and $\mathcal{D} = \{\emptyset, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, X\}$. Show that $\mathcal{D}$ is not an algebra.

(Note: It can be shown than the collection $\mathcal{D}$ is closed under the formation of complements and finite disjoint unions. Hence such a collection need not be an algebra.)

5) Suppose $X \in \mathcal{C}$ and $A, B \in \mathcal{C} \Rightarrow A - B = A \cap B^c \in \mathcal{C}$. Show $\mathcal{C}$ is an algebra.

Hint: Be sure to note that $X \in \mathcal{C}$ means that $\mathcal{C}$ is nonempty.