Math 501 HW 4 Spring 2025. Due Friday, Feb. 14.

Exam 1 review may be useful. For the quiz, the exam 1 review and qual problems from the course website may be useful. 3 sheets of notes for the quiz.

1) Let $\mathcal{C}_O$ be the class of all open real sets. Let $\mathcal{C}_C$ be the class of all closed real sets. Let $\mathcal{C}_{CI}$ be the class of all closed intervals in $\mathbb{R} = X$. Then the Borel σ-algebra $\mathbb{B}(\mathbb{R}) = \sigma(\mathcal{C}_O)$. Prove that $\sigma(\mathcal{C}_{CI}) \subseteq \sigma(\mathcal{C}_C)$.

Hint: If $\mathcal{C} \subseteq \sigma$-algebra $\mathcal{F}$, then $\sigma(\mathcal{C}) \subseteq \mathcal{F}$. Follow the examples done for point 53) in the class notes.

2) Let $A = \{1/2, 1/3, ..., 1/n...\}$ (so $A = \{a_1, a_2, ...\}$ with $a_n = 1/(n+1)$). Is the set A closed? Explain briefly.
Hint: Exam 1 review 45) is useful.

3) Referring to problem 1, let $\mathcal{C}_{OI}$ be the class of all open intervals in $\mathbb{R} = X$. Prove that $\sigma(\mathcal{C}_O) = \sigma(\mathcal{C}_{OI})$.

Hint: This is an examples done for point 53) in the class notes. Also see problem number 2.1) from the PhD qual problems with a link from the Math 501 webpage: (http://parker.ad.siu.edu/Olive/zM501qualprob.pdf).

4) Simplify the following set where $a_i < b_i$. $B = \cup_{n=1}^{\infty} B_n = \cup_{n=1}^{\infty}([-n, a_i] \cup [b_i, n])$.

Hint: You should get $B = C \cup D$ where C and D are intervals.

Note: Let $B_n = ([-n, a_i] \cup [b_i, n])$. Then $B_n \uparrow B = C \cup D$

5) Let $X = [0,1]$ and consider the Borel $\sigma-$field $\mathbb{B}([0,1]) = \sigma(\mathcal{C})$ where $\mathcal{C}$ is the class of open intervals in [0,1]. Prove that $\{0,1\} \in \mathbb{B}([0,1])$.