Math 501 HW 7 Spring 2025. Due Friday, March 21.

Exam 2 review may be useful.

1) Let simple function $\phi = \sum_{i=1}^{n} a_i \ \chi_{E_i}$ vanish outside a set of finite measure (with $m(E_i) < \infty$ for each i). If measurable sets A and B are disjoint, prove that $\int_{A \cup B} \phi = \int_A \phi + \int_B \phi$.

Hint: Under the conditions of 1), $\int_E \phi = \int \phi \ \chi_E$. Use linearity and problem 1 b) from homework 6 with $E = A \cup B$.

2) Let ϕ and ψ be simple functions which vanish outside a set E of finite measure. Prove that if $\phi \geq \psi$ ae, then $\int_E \phi \geq \int_E \psi$.

Hint: Note that $(\phi - \psi)$ is a simple function that is nonnegative on E ae. Let $\phi(x) - \psi(x) = \sum_{i=1}^{n} a_i \chi_{E_i}(x)$ be the canonical representation of $(\phi - \psi)$. (Also note that $(\phi - \psi) = (\phi - \psi) \ \chi_E$ on E since $\phi(x) - \psi(x) = 0 - 0 = 0$ for $x \notin E$.) Note that if $a_i < 0$, then $m(E_i) = 0$.

3) Let A and B be measurable sets. Under what conditions does $m(A - B) = m(A) - m(B)$?

4) Suppose $m(E) < \infty$ and $\phi(x) = c$ for all $x \in E$ where c is a constant. Find $\int c\chi_E$.

5) Let $A, B \in \mathcal{F}_M$, $m(A) = 3$, and $m(A \cap B) = 1$. Let $\phi = 2\chi_A$. Let $\int f = 8$. For a) − d) calculate

a) $\int \chi_A$
b) $\int \phi$
c) $\int (2f + 4\phi)$
d) $m(A - B)$.
e) Show $m(B) \geq 1$.