Math 501 Quiz 1 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Simplify the following sets. Answers might be $(a,b), [a,b), (a,b], [a,b], [a,a] = \{a\}, (a,a) = \emptyset$. See if 0 is always in or never the set I_n where $I_n = \left(0, \frac{1}{n}\right)$ or $I_n = \left[0, \frac{1}{n}\right)$.

i) $\bigcap_{n=1}^{\infty} \left(0, \frac{1}{n}\right) =$ $\phi = (0,0)$

ii) $\bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right) =$ $[0,0] = \{0\}$

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2) Let $f : D \to E$ be a function where D is the *domain* of f.
a) Let $B \subseteq E$. What is the **inverse image** of $B = f^{-1}[B] =$

$= \{x \in D : f(x) \in B\}$,

b) Let $A \subseteq D$. What is the **image** under f of $A = f[A] =$

$\{y \in E : y = f(x) \text{ for some } x \in A\}$

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3) A DeMorgan's law can be written as $\left[\bigcap_{k=n}^{N} A_k\right]^c = \bigcup_{k=n}^{N} A_k^c$ where $N \geq n$, n is a positive integer, and $N = \infty$ is allowed.

i) Find $\left[\bigcup_{k=n}^{N} A_k\right]^c$ using the above law (and complementation).

$$= \boxed{\bigcap_{k=n}^{N} A_k^c}$$

ii) Let $D = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k^c = \bigcap_{n=1}^{\infty} C_n^c$ where $C_n^c = \bigcup_{k=n}^{\infty} A_k^c$.

Use DeMorgan's law to find $\left[\bigcap_{n=1}^{\infty} C_n^c\right]^c$.

$$= \boxed{\bigcup_{n=1}^{\infty} C_n}$$

iii) Find C_n.

$$= [C_n^c]^c = \left[\bigcup_{k=n}^{\infty} A_k^c\right]^c = \boxed{\bigcap_{k=n}^{\infty} A_k}$$

iv) Use the above results to show
$D^c = \left[\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k^c\right]^c = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$.

with practice

$$= \bigcup_{n=1}^{\infty} \left[\bigcup_{k=n}^{\infty} A_k^c\right]^c$$

$$= \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$$

$$D^c = \left[\bigcap_{n=1}^{\infty} C_n^c\right]^c = \bigcup_{n=1}^{\infty} C_n$$

$$= \boxed{\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k}$$

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