Math 501 Quiz 2 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) If a set B is a sequence of unions and intersections of sets, then DeMorgan's law (applied iteratively) says change each union to an intersection, change each intersection to a union, and take the complement of the final set. Use this method to write down the complement B^c of the following sets (quickly).

a) $B = \cap_{n=1}^{\infty} \cup_{k=n}^{\infty} A_k$. So
$B^c =$ $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k^c$

b) $B = \cup_{n=1}^{\infty} \cap_{k=n}^{\infty} A_k^c$. So
$B^c =$ $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

c) $B = \cup_{i_1=1}^{\infty} \cap_{i_2=i_1}^{\infty} \cdots \cup_{i_k=i_{k-1}}^{\infty} A_{i_k}^c$ So
$B^c =$

$\bigcap_{i_1=1}^{\infty} \bigcup_{i_2=i_1}^{\infty} \cdots \bigcap_{i_k=i_{k-1}}^{\infty} A_{i_k}$

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2) Let $a < b$ and let $I = \bigcup_{n=1}^{\infty} \left[a + \frac{1}{n}, b - \frac{1}{n}\right] = \bigcup_{n \in \mathbb{N}} \left[a + \frac{1}{n}, b - \frac{1}{n}\right] = \bigcup_{n=m}^{\infty} \left[a + \frac{1}{n}, b - \frac{1}{n}\right]$

where m is the smallest positive integer such that $a + \frac{1}{m} \leq b - \frac{1}{m}$ since $[c, d] = \emptyset$ if $c > d$. I is equal to an interval. Find that interval.

$I = \boxed{(a, b)}$

($A_n = \left[a + \frac{1}{n}, b - \frac{1}{n}\right] \uparrow I$

a and b are never in A_n

$a+\varepsilon$ and $b-\varepsilon$ are eventually in I

for large enough n where $\varepsilon > 0$

and $a+\varepsilon < b-\varepsilon$!

$a < a+\frac{1}{n}$ and $b > b-\frac{1}{n}$ $\forall n \Rightarrow a,b \notin I$)

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3) Let Λ be an arbitrary nonempty index set, and for $\lambda \in \Lambda$, let $\mathcal{F}_\lambda$ be a $\sigma-$~~field~~ algebra on X. Prove that $\mathcal{F} = \bigcap_{\lambda \in \Lambda} \mathcal{F}_\lambda$ is a $\sigma-$~~field~~ algebra on X.

Hint: To prove that $\mathcal{F}$ is a $\sigma-$algebra, show

0) $\mathcal{F}$ is nonempty. Often this is done by showing that $X \in \mathcal{F}$.

i) $A_1, A_2, \ldots \in \mathcal{F} \Rightarrow \cup_{i=1}^{\infty} A_i \in \mathcal{F}$.

ii) $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$.

0) $X \in \mathcal{F}_\lambda \ \forall \lambda$, $\therefore X \in \mathcal{F}$.

i) $A_1, A_2 \ldots \in \mathcal{F} \Rightarrow$

$A_1, A_2, \ldots \in \mathcal{F}_\lambda \ \forall \lambda$

$\Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}_\lambda \quad \forall \lambda$

$\Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$

ii) $A \in \mathcal{F} \Rightarrow A \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow$

$A^c \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow A^c \in \mathcal{F}$.

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