Math 501   Quiz 5 Spring 2025   Name____________________

YOU ARE BEING GRADED FOR WORK

Q

1) a) Give the definition of a measurable set (where $X = \mathbb{R}$).

E is measurable if for any set $A \, (\subseteq \mathbb{R})$

$$m^*(A) = m^*(A \cap E) + m^*(A \cap E^c).$$

b) Suppose V is a nonmeasurable set. Then there exists $A \subseteq \mathbb{R}$ such that the equation in a) does not hold with $E = V$. Explain why this result proves that the outer measure m^* is not finitely additive on the σ-algebra $\mathbb{P}$ of all subsets of $\mathbb{R}$. $\mathbb{P}(\mathbb{R})$

V is nonmeasurable

$\therefore \exists A \; (=A_V) \ni$

$$m^*(A) \neq m^*(A \cap V) + m^*(A \cap V^c)$$

$$\text{but } A = \underbrace{(A \cap V) \cup (A \cap V^c)}_{\text{disjoint}}$$

Thus finite additivity fails

(for sets A, $A \cap V$, and $A \cap V^c$).

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2) Let $\mathbb{Q}$ be the set of rational number in $\mathbb{R}$. Find $m(\mathbb{Q})$.

$= \boxed{0}$ since $\mathbb{Q}$ is countable.

16 3) Let A be the set of irrational numbers in $\mathbb{R}$. Find $m(A)$.

$m(A \cup \mathbb{Q}) = m(\mathbb{Q}^c \cup \mathbb{Q}) = m(\mathbb{R}) =$

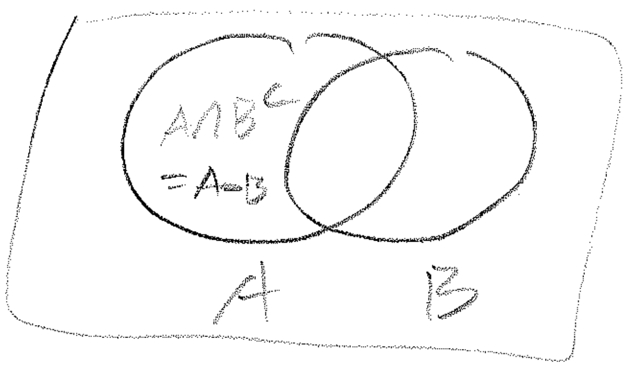
$\infty \underset{\uparrow \text{finite additivity}}{=} m(A) + m(\mathbb{Q}) = m(A) = \infty$

16 4) Prove the set A in 3) is uncountable (with 1 sentence).

A is uncountable since $m(A) > 0$.

17 5) Let A and B be measurable. Then $A \cup B = (A \cap B^c) \cup B$. Use finite additivity to find $m(A \cup B) = m(\ (A \cap B^c) \cup B)$.

$= \boxed{m(A \cap B^c) + m(B)}$



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