Math 501 Quiz 6 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Give the definition of a measurable function. $f: D \to \mathbb{R}^*$ is a measurable function if D is measurable and if for each $a \in \mathbb{R}$, $\{x \in D : f(x) > a\} \in \mathcal{F}_M$ (is a measurable set).

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2) Let $f, g \in \mathcal{L}(D)$. Prove $\max(f, g) \in \mathcal{L}(D)$.

$\forall t$, $\{x \in D : \max(f(x), g(x)) \leq t\} =$

$\{x \in D : f(x) \leq t\} \cap \{x \in D : g(x) \leq t\} \in \mathcal{F}_M$

or $\forall t$ $\{x \in D : \max(f(x), g(x)) > t\}$

$= \{x \in D : f(x) > t\} \cup \{x \in D : g(x) > t\} \in \mathcal{F}_M$

(since $\max(f(x), g(x)) \leq t$ if both $f(x) \leq t$ and $g(x) \leq t$)

$\cap$ both

$\cup$ at least one

go on!!

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3) Let $D = \mathbb{R}$. Then $f : \mathbb{R} \to Y$ is a measurable function ($f \in \mathcal{L}(\mathbb{R})$) if for each

A) $t \in \mathbb{R}$, $f^{-1}[(t, \infty)] = \{x : f(x) > t\} \in \mathcal{F}_M$. Prove that $f \in \mathcal{L}(\mathbb{R})$ if $f^{-1}[(-\infty, t]] =$

B) $\{x : f(x) \leq t\} = \{x \in \mathbb{R} : f(x) \leq t\} \in \mathcal{F}_M$ for each $t \in \mathbb{R}$.

If $\{x : f(x) \leq t\} \in \mathcal{F}_M$ $\forall t$, then

$\{x : f(x) \leq t\}^c = \{x : f(x) > t\} \in \mathcal{F}_M$ $\forall t$,

$\forall t$ or -4

want to show $B \Rightarrow A$

not $A \Rightarrow B$ $\}$ -7

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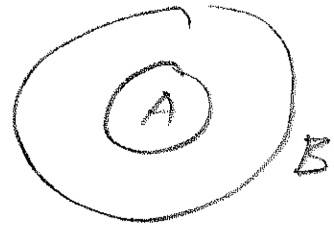
[illegible]

4) Let $A, B, A_i \in \mathcal{F}_M$ be measurable sets. You may assume finite additivity: if $A_1, ..., A_n$ are disjoint, then $m(\cup_{i=1}^n A_i) = \sum_{i=1}^n m(A_i)$. If $A \subseteq B$ and $m(B) < \infty$, prove $m(B - A) = m(B) - m(A)$.

so $m(A) < \infty$ too

$$B = \underbrace{A \cup (B - A)}_{\text{disjoint}}$$



$\therefore$ $m(B) = m(A) + m(B - A)$.

Thus $m(B - A) = m(B) - m(A)$

$(\neq \infty - \infty$ since $m(A) \leq m(B) < \infty)$

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