Math 501 Quiz 7 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Let $A, B \in \mathcal{F}_M$, $m(A) = 5$, and $m(A \cap B) = 3$. Let $\phi = 7\chi_A$. Let $\int f = 4$. For a) – d) calculate

a) $\int \chi_A$ $= m(A) = \boxed{5}$

b) $\int \phi$ $= 7\, m(A) = 7(5) = \boxed{35}$

c) $\int (3f + 2\phi)$ $= 3\int f + 2\int \phi$

$= 3(4) + 2(35) = \boxed{82}$

d) $m(A - B)$. $A = (A \cap B) \cup (A - B)$ disjoint

so $m(A - B) = m(A) - m(A \cap B) = 5 - 3 = \boxed{2}$

e) Show $m(B) \geq 3$.

$A \cap B \subseteq B \therefore m(B) \geq m(A \cap B) = 3$

if $m(A \cap B) < \infty$

$m(A \cup B) = m(A) + m(B) - m(A \cap B)$

$m(B) = \underbrace{m(A \cup B) - m(A)}_{\geq 0 \text{ since } A \subseteq A \cup B} + m(A \cap B)$ so $m(A \cup B) \geq 5$

50

2) Let $A_i \in \mathcal{F}_M$ be measurable sets. Let $B_i = A_i - (\cup_{j=1}^{i-1} A_j)$ for $i = 1, 2, 3, \ldots$ with $B_1 = A_1$, $B_2 = A_2 \cap A_1^c$ etc. Then $\cup_{i=1}^{\infty} A_i = \cup_{i=1}^{\infty} B_i$ where the B_i are pairwise disjoint. Find $m(\cup_{i=1}^{\infty} A_i)$.

$$= m\left(\bigcup_{i=1}^{\infty} B_i\right) = \boxed{\sum_{i=1}^{\infty} m(B_i)}$$

by countable additivity

10

3) Let $f, g \in \mathcal{L}(D)$. Prove $\min(f, g) \in \mathcal{L}(D)$.

$\forall t$, $\{x \in D : \min(f(x), g(x)) \le t\} =$

$\{x \in D : f(x) \le t\} \cup \{x \in D : g(x) \le t\} \in \mathfrak{F}_m$

(or $\forall t$, $\{x \in D : \min(f(x), g(x)) \ge t\} =$

$\{x \in D : f(x) \ge t\} \cap \{x \in D : g(x) \ge t\} \in \mathfrak{F}_m$)

20

4) Let $D = \mathbb{R}$. Then $f : \mathbb{R} \to Y$ is a measurable function ($f \in \mathcal{L}(\mathbb{R})$) if for each
A $t \in \mathbb{R}$, $f^{-1}[(t, \infty)] = \{x : f(x) > t\} \in \mathcal{F}_M$. Assume that f is a measurable function if for
B each $t \in \mathbb{R}$, $f^{-1}[[t, \infty)] = \{x : f(x) \ge t\} \in \mathcal{F}_M$. Prove that $f \in \mathcal{L}(\mathbb{R})$ if $f^{-1}[(-\infty, t)] =$
C $\{x : f(x) < t\} = \{x \in \mathbb{R} : f(x) < t\} \in \mathcal{F}_M$ for each $t \in \mathbb{R}$.

If $\{x : f(x) < t\} \in \mathfrak{F}_m$ $\forall t$, then

$\{x : f(x) < t\}^c = \{x : f(x) \ge t\} \in \mathfrak{F}_m$ $\forall t$,

20