take home due F

Math 501 Quiz 3 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Let $A = (0, \infty)$. Is χ_A L. integrable? Explain briefly.

no, $\int \chi_A = m(A) = \infty$

(need $\int \chi_A < \infty$)

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2) a) State the Bounded Convergence Theorem. Let $E \in \mathcal{F}_m$, $m(E) < \infty$, and $f_n \in \mathcal{F}(E)$, If $\exists M > 0 \ni |f_n(x)| \leq M \; \forall n \in \mathbb{N}$ and $\forall x \in E$ and if $\lim_{n \to \infty} f_n(x) = f(x) \; \forall x$ in E, then

$$\lim_{n \to \infty} \int_E f_n = \int_E f.$$

b) Use a) to find $\lim_{n\to\infty} \int_0^1 \frac{dx}{(1+x^2)^n}$. Hint: Find the integrand $f_n = f_n(x)$ and find $\lim_{n\to\infty} f_n = f = f(x)$. on $E = (0,1)$.

$f_n(x) = \frac{1}{(1+x^2)^n}$ $\quad 1+x^2 > 0$ on $(0,1)$

$|f_n(x)| \leq 1$ on E

$\therefore (1+x^2)^n \to \infty$ as $n \to \infty$.

$\therefore f_n(x) \to f(x) = 0$ on $(0,1)$.

$\therefore \lim_{n\to\infty} \int_0^1 \frac{dx}{(1+x^2)^n} dx =$

$\lim_{n\to\infty} \int_0^1 f_n = \int_0^1 f = \int_0^1 0 \, dx = \boxed{0}$

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3) What theorem or lemma is used to prove the Mononone Convergence Theorem?

Fatou's lemma

4) Suppose

$f(x) = -1$ a.e. on $[0,1]$.

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \cap [0,1] \\ -1, & x \notin \mathbb{Q} \cap [0,1] \end{cases}$$

a) Does the Riemann integral $\int_0^1 f(x)dx$ exist?

no $f(x)$ is discontinuous on $[0,1]$ and $m([0,1]) = 1 > 0$.

Set of discontinuities does not have measure 0 $\therefore$ or f is not continuous a.e. on E

b) Find $\int_0^1 |f(x)|dx$.

$$= \int_0^1 1 dx = x\Big|_0^1 = \boxed{1}$$

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