Math 501 Quiz 9 Spring 2025 Name____________________

YOU ARE BEING GRADED FOR WORK

1) The f(x) = sinc(x) function has sinc(x) = $\sin(x)/x$ for $x \neq 0$ and sinc(0) = 1. It is known that the improper Riemann integral $\int_0^\infty f(x)dx = \pi/2$. It is also known that the Riemann or Lebesgue integral $\int_0^\infty |f(x)|dx = \infty$. Is f (Lebesgue) integrable?

no need $\int |f| < \infty$

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2) Let the a_i be real numbers, and the f_i integrable functions on measurable set E. (Integrable functions are measurable.) Is it true that $\displaystyle\int_E \sum_{i=1}^n a_i f_i = \sum_{i=1}^n a_i \int_E f_i$? Explain briefly.

Yes. Let $g_i = a_i f_i$. Then by linearity $\int_E \sum_{i=1}^n g_i = \sum_{i=1}^n \int_E g_i = \sum_{i=1}^n \int_E a_i f_i$

$= \sum_{i=1}^n a_i \int_E f_i$.

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3) Let $f(x) = 0$ for all $x \in E \in \mathcal{F}_M$. Let g be any nonnegative function that is integrable over E. Hence $g \geq f$ on E. Prove that $\int_E g \geq 0$.

By monotonicity, $\int_E g \geq \int_E f = \int_E 0 = 0$

$\int 0 \chi_E = 0\, m(E) = 0$.

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4) If a function f is monotone on $[a, b]$, then the total variation of the function is $T = V_a^b(f) = |f(b) - f(a)| < \infty$. Hence f is of bounded variation over $[a, b]$. Nondecreasing and nonincreasing functions are monotone. Increasing functions are nondecreasing and decreasing functions are nonincreasing. If f is nondecreasing on $[a, b]$, then $-f$ is nonincreasing on $[a, b]$. Let f be differentiable on open interval I. If $f'(x) > 0$ on I, then f is an increasing function on I. If $f'(x) < 0$ on I, then f is an decreasing function on I. If $f(x) = c$ on I where c is a constant, then f is both a nondecreasing and a nonincreasing function on I. A similar result holds for $[a, b]$ if $I = (a, b)$ and f is continuous on $[a, b]$. Let $f(x) = x^2$ on $[0, 1]$.

a) Find $T = V_0^1(f)$. CT so $T = |f(1) - f(0)| = 1^2 - 0^2 = \boxed{1}$

b) Is f is of bounded variation over $[0, 1]$?

Yes since $T = 1 < \infty$

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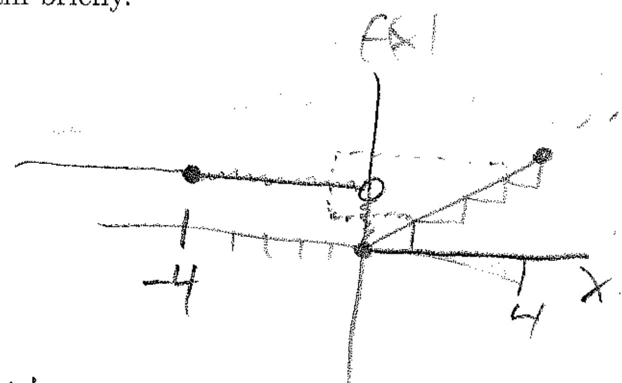
5) Suppose the function f satisfies $f(x) = 1$ for $x < 0$ and $f(x) = x$ for $x \geq 0$. Is f of bounded variation on $[-4, 4]$? Explain briefly.



Yes, $T = 0 + 1 + 4 - 0 = 5$.

If π does not have an $x_i = 0$, adding $x_i = 0$ to π increases $V_{-4}^{4}(f, \pi)$

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