1) Let $\mathcal{A}$ be a class of subsets of X. The $\sigma-$algebra generated by $\mathcal{A}$, denoted by $\sigma(\mathcal{A})$, is the intersection of all $\sigma-$algebras containing $\mathcal{A}$. Prove that $\sigma(\mathcal{A})$ is a $\sigma-$algebra.

Proof. i) Let $\sigma(\mathcal{A}) = \cap_{\lambda \in \Lambda} \mathcal{F}_\lambda$ where Λ is the collection of $\sigma-$algebras $\mathcal{F}_\lambda$ that contain $\mathcal{A}$. Then Λ is nonempty since the $\sigma-$algebra of all subsets of X is in Λ.

ii) If $A_1, A_2, ... \in \sigma(\mathcal{A})$, then $A_1, A_2, ... \in \mathcal{F}_\lambda$ for each $\lambda \in \Lambda$. Hence $\cup_{i=1}^\infty A_i \in \mathcal{F}_\lambda$ for each $\lambda \in \Lambda$. Thus $\cup_{i=1}^\infty A_i \in \sigma(\mathcal{A})$.

iii) If $A \in \sigma(\mathcal{A})$, then $A \in \mathcal{F}_\lambda$ for each $\lambda \in \Lambda$. Hence $A^c \in \mathcal{F}_\lambda$ for each $\lambda \in \Lambda$. Thus $A^c \in \sigma(\mathcal{A})$. $\square$