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Math 580  HW 10 Spring 2022. Due Wednesday, April 13.
Quiz 9 on MLE, MSE, UMVUE, I_1(theta), FCRLB
This homework has 1 page, 3 problems.
Final: Monday, May 2, time 8-10 AM in the morning
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1. (6.7) Let $X_1, ..., X_n$ be independent, identically distributed $N(\mu, 1)$ random variables where μ is unknown and $n \geq 2$. Let t be a fixed real number. Then the expectation

$$E_\mu(I_{(-\infty,t]}(X_1)) = P_\mu(X_1 \leq t) = \Phi(t - \mu)$$

for all μ where $\Phi(x)$ is the cumulative distribution function of a $N(0, 1)$ random variable.

a) Show that the sample mean $\overline{X}$ is a sufficient statistic for μ.

b) Explain why (or show that) $\overline{X}$ is a complete sufficient statistic for μ.

c) Using the fact that the conditional distribution of X_1 given $\overline{X} = \overline{x}$ is the $N(\overline{x}, 1 - 1/n)$ distribution where the second parameter $1 - 1/n$ is the variance of conditional distribution, find

$$E_\mu(I_{(-\infty,t]}(X_1)|\overline{X} = \overline{x}) = E_\mu[I_{(-\infty,t]}(W)]$$

where $W \sim N(\overline{x}, 1 - 1/n)$. (Hint: your answer should be $\Phi(g(\overline{x}))$ for some function g.)

d) What is the uniformly minimum variance unbiased estimator for $\Phi(t - \mu)$?

2. (7.2) Let $X_1, ..., X_n$ be a random sample from the distribution with pdf

$$f(x|\theta) = \frac{x^{\theta-1}e^{-x}}{\Gamma(\theta)}, \ x > 0, \ \theta > 0.$$

For a) and b) do not put the rejection region into useful form.

a) Use the Neyman Pearson Lemma to find the UMP size α test for testing $H_o : \theta = 1$ vs $H_1 : \theta = \theta_1$ where θ_1 is a fixed number greater than 1.

b) Find the uniformly most powerful level α test of

$$H_0\text{: } \theta = 1 \text{ versus } H_1\text{: } \theta > 1.$$

Justify your steps. (Hint: Use the statistic in part a).

3. (7.3) Let $H_o : X_1, ..., X_n$ are iid $U(0, 10)$ and $H_1 : X_1, ..., X_n$ are iid $U(4, 7)$. Suppose you had a sample of size $n = 1000$. How would you decide which hypothesis is true?