`Math 580  HW 11 Spring 2022. Due Wednesday, April 20.`

Exam 3 is Wednesday, April 27. The Final is Monday, May 2, time 8-10 in the MORNING. HW is one page, 6 problems.

1) Suppose that we have two independent samples: $X_1, ..., X_n$ are exponential (θ), and $Y_1, ..., Y_m$ are exponential (μ). Find the likelihood ratio test for $H_0 : \theta = \mu$ versus $H_1 : \theta \neq \mu$. Do not put the rejection region into useful form.

2) Let $Y_1, ..., Y_{10}$ be iid Ber(ρ). Find the UMP test for $\alpha = 0.0547$ of $H_0 : \rho = 0.5$ versus $H_1 : \rho = 0.25$. Put the test in useful form.

3) Let $Y_1, ..., Y_n$ be iid Poisson(θ). Find the UMP test of $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$ using the Neyman Pearson lemma. Your final test statistic should have a Poisson distribution when H_0 is true.

4) (7.1) Let $X_1, ..., X_n$ be iid $N(\mu, \sigma^2)$, $\sigma^2 > 0$. Let $\Theta_o = \{(\mu_o, \sigma^2) : \mu_o \text{ fixed}, \sigma^2 > 0\}$ and let $\Theta = \{(\mu, \sigma^2) : \mu \in \Re, \sigma^2 > 0\}$. Consider testing $H_o : \theta = (\mu, \sigma^2) \in \Theta_o$ vs H_1: not H_o. The MLE $\hat{\theta} = (\hat{\mu}, \hat{\sigma}^2) = (\overline{X}, \frac{1}{n}\sum_{i=1}^n (X_i - \overline{X})^2)$ while the restricted MLE is $\hat{\theta}_o = (\hat{\mu}_o, \hat{\sigma}_o^2) = (\mu_o, \frac{1}{n}\sum_{i=1}^n (X_i - \mu_o)^2)$.

a) Show that the likelihood ratio statistic

$$\lambda(\boldsymbol{x}) = (\hat{\sigma}^2/\hat{\sigma}_o^2)^{n/2} = [1 + \frac{n(\overline{x} - \mu_o)^2}{\sum_{i=1}^n (x_i - \overline{x})^2}]^{-n/2}.$$

b) Show that H_o is rejected iff $|\sqrt{n}(\overline{X} - \mu_o)/S| \geq k$ and find k if $n = 11$ and $\alpha = 0.05$. (Hint: show that H_o is rejected iff $n(\overline{X} - \mu_o)^2 / \sum_{i=1}^n (X_i - \overline{X})^2 \geq c$, then multiply both sides by a constant such that the left hand side has a $(t_{n-1})^2$ distribution. Use the fact that $\frac{\overline{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$ under H_0. Use the t-table on the handout to find k.)

5) (8.2) Let $X_1, ..., X_n$ be iid from a normal distribution with unknown mean μ and known variance σ^2. Let $\overline{X} = \frac{\sum_{i=1}^n X_i}{n}$. Find the limiting distribution of $\sqrt{n}(\overline{X}^3 - c)$ for an appropriate constant c.

6) (8.3ac) Let $X_1, ..., X_n$ be a random sample from a population with pdf

$$f(x) = \begin{cases} \frac{\theta x^{\theta - 1}}{3^\theta} & 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

The method of moments estimator for θ is $T_n = \dfrac{\overline{X}}{3 - \overline{X}}$.

a) Find the limiting distribution of $\sqrt{n}(T_n - \theta)$ as $n \to \infty$.

c) Find a consistent estimator for θ and show that it is consistent.