```
Math 580    HW 3 Spring 2022.  Due Wednesday, Feb. 2.
Problems 1-7. Two pages. Exam 1 is Wednesday, Feb. 9.
```

1. Suppose $X_i|P_i \sim$ Bernoulli($\mathrm{P_i}$) and $P_i \sim beta(\delta, \nu)$ for $i = 1, ..., n$. Assume $Y = \sum_{i=1}^n X_i$ where the X_i are independent.

a) Find $E(X_i)$. b) Find $V(X_i)$.
c) Find $E(Y)$. d) Find $V(Y)$.

2. A pdf is defined by $f(x, y) = c(x + 2y)$ for $0 < y < 1$ and $0 < x < 2$.

a) Find c.

b) Find the marginal distribution of X. Include the support.

c) Find the joint cdf of X and Y on the support. Include the support.

d) Are X and Y independent? Explain briefly.

3. Suppose that the joint probability mass function $f(y_1, y_2)$ of Y_1 and Y_2 is given in the following table.

$f(y_1, y_2)$		y_2 = 1	2	3
	2	1/12	1/6	1/12
y_1	3	1/6	0	1/6
	4	0	1/3	0

a) Why are Y_1 and Y_2 dependent?

b) Give a pmf table $f(u, v)$ for independent random variables U and V that have the same marginals as Y_1 and Y_2.

4. (2.75) Find the pmf of $Y = X^2 + 4$ where the pmf of X is given below.

```
-----------------------------------------------
     X      | -2       -1       0     1     2
 probability | 0.1      0.2     0.4   0.2   0.1
-----------------------------------------------
```

5. (2.16a) If Y is power $POW(\lambda)$, then the pdf of Y is

$$f(y) = \frac{1}{\lambda} y^{\frac{1}{\lambda} - 1},$$

where $\lambda > 0$ and $0 < y < 1$. Show that $W = -\log(Y)$ is an exponential (λ) random variable.

Note: 2.16a means only do part a). Hint: you may assume that $0 < y < 1$ since the endpoints do not matter for a pdf. Do not forget the support.

6. (2.9b) If Y is a normal $N(\mu, \sigma^2)$ random variable, show that $W = e^Y$ is a lognormal $LN(\mu, \sigma^2)$ random variable.

Hint: get the pdfs from chapter 10: If Y has a lognormal distribution, $Y \sim LN(\mu, \sigma^2)$, then the pdf of Y is

$$f(y) = \frac{1}{y\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(\log(y) - \mu)^2}{2\sigma^2}\right)$$

where $y > 0$ and $\sigma > 0$ and μ is real.

7. (2.67abcd) (This notation means do parts a), b), c) and d) but do not do part e).) Suppose that X_1 and X_2 have a joint pdf given by

$$f(x_1, x_2) = 3(x_1 + x_2)I(0 < x_1 < 1)I(0 < x_2 < 1)I(0 < x_1 + x_2 < 1).$$

Consider the transformation $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

a) Find the Jacobian J for the transformation.

b) Find the support $\mathcal{Y}$ of Y_1 and Y_2.

c) Find the joint density $f_{Y_1,Y_2}(y_1, y_2)$.

d) Find the marginal pdf $f_{Y_1}(y_1)$.

Hint: use indicators to find the support $\mathcal{Y}$. To do d), 4 lines of the form $y_2 = a + by_1$ and 2 lines of the form $y_1 = c$ determine $\mathcal{Y}$. Sketch $\mathcal{Y}$ then find the marginal. Do not forget the support for part d).