`Math 580    HW 4 Spring 2022.  Due Wednesday, Feb 16.`

`EXAM 1 is on Wednesday, Feb. 9. NO NOTES, but bring a  calculator.`

`My webpage (http://parker.ad.siu.edu/Olive/infer.htm) has a link for statistical inference.`

For the following 4 problems, follow the examples (for problems 2.4 and 2.6) using Theorem 2.16e done in class. (Do not use Theorem 2.17 or Theorem 2.18.)

1. (2.1) Let $X_1, ..., X_n$ be independent random variables with $X_i \sim$ Poisson(λ_i). Let $W = \sum_{i=1}^n X_i$. Find the mgf of W and find the distribution of W.

2. (2.2) Let $X_1, ..., X_n$ be independent random variables with $X_i \sim$ Bernoulli(ρ). Let $W = \sum_{i=1}^n X_i$. Find the mgf of W and find the distribution of W.

3. (2.3) Let $X_1, ..., X_n$ be independent random variables with $X_i \sim$ exponential (λ). Let $W = \sum_{i=1}^n X_i$. Find the mgf of W and find the distribution of W.

4. (2.5) Let $X_1, ..., X_n$ be independent random variables with $X_i \sim$ negative binomial (1, ρ). Let $W = \sum_{i=1}^n X_i$. Find the mgf of W and find the distribution of W.

5. (2.64) Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4 \left(\begin{pmatrix} 49 \\ 100 \\ 17 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 & 1 & -1 & 0 \\ 1 & 6 & 1 & -1 \\ -1 & 1 & 4 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix} \right).$$

a) Find the distribution of X_2.

b) Find the distribution of $(X_1, X_3)^T$.

c) Which pairs of random variables X_i and X_j are independent?

d) Find the correlation $\rho(X_1, X_3)$.

6. (2.66) Let $\sigma_{12} = Cov(Y, X)$ and suppose Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2 \left(\begin{pmatrix} 15 \\ 20 \end{pmatrix}, \begin{pmatrix} 64 & \sigma_{12} \\ \sigma_{12} & 81 \end{pmatrix} \right).$$

a) If $\sigma_{12} = 10$ find $E(Y|X)$.

b) If $\sigma_{12} = 10$, find Var$(Y|X)$.

c) If $\sigma_{12} = 10$, find $\rho(Y, X)$, the correlation between Y and X.