Math 580   HW 5 Spring 2022. Due Wednesday, Feb. 23.
Exam 2 is Wednesday, March 23.

Problems 1), 2), ..., 7) are closely related to problems 3.1-3.4. Follow the chi(p, σ) example done in class with both parameters unknown. Also see examples in chapters 3 and 10. **One page, 11 problems**.

For each of the following families in problems 1), ..., 7): a) show that the family is an exponential family, b) find the natural parameter space, c) determine whether the family is a regular exponential family or not.

1) The $N(\mu, \sigma^2)$ family with $\sigma > 0$ known.

2) The $N(\mu, \sigma^2)$ family with μ known and $\sigma > 0$.
(See problem 9 = 3.12 for a common error.)

3) The gamma (ν, λ) family with ν known.

4) Show that the gamma (ν, λ) family with λ known.

5) The beta (δ, ν) family with both δ and ν unknown.

6) The Poisson (θ) family.

7) The negative binomial family with r known and $0 < \rho < 1$.
If $Y \sim$ NB(r, ρ), then the pmf of Y is

$$f(y) = P(Y = y) = \binom{r+y-1}{y} \rho^r (1-\rho)^y$$

for $y = 0, 1, \ldots$ where $0 < \rho < 1$.

8) (3.6) Determine whether the Pareto(σ, λ) family is an exponential family or not.
If $Y \sim$ PAR(σ, λ), then the pdf of Y is

$$f(y) = \frac{\frac{1}{\lambda}\sigma^{1/\lambda}}{y^{1+1/\lambda}}$$

where $y \geq \sigma$, $\sigma > 0$, and $\lambda > 0$.

9) (3.12) Suppose that X has a N(μ, σ^2) distribution where $\sigma > 0$ and μ **is known**. Then

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\mu^2/(2\sigma^2)} \exp[-\frac{1}{2\sigma^2}x^2 + \frac{1}{\sigma^2}\mu x].$$

Let $\eta_1 = -1/(2\sigma^2)$ and $\eta_2 = 1/\sigma^2$. Why is this parameterization not the regular exponential family parameterization? (Hint: show that η_1 and η_2 satisfy a linearity constraint.)

10) If $Y_1, ..., Y_n$ are iid $N(\mu, \sigma^2)$, find $V(S^2)$.

11) (4.2) Let $X_1, ..., X_n$ be iid exponential(λ) random variables. Use the Factorization Theorem on p. 108 to show that $T(\boldsymbol{X}) = \sum_{i=1}^n X_i$ is a sufficient statistic for λ.