```
Math 580    HW 6 Spring 2022.  Due Wednesday, March 2.
Exam 2 is Wednesday, March 23.  One page, 6 problems
```

1) (4.3): Let $X_1, ..., X_n$ be iid from a regular exponential family with pdf

$$f(x|\boldsymbol{\eta}) = h(x)b(\boldsymbol{\eta}) \exp[\sum_{i=1}^{k} \eta_i t_i(x)].$$

Let $\boldsymbol{T}(\boldsymbol{X}) = (T_1(\boldsymbol{X}), ..., T_k(\boldsymbol{X}))$ where $T_i(\boldsymbol{X}) = \sum_{j=1}^{n} t_i(X_j)$.

a) Use the Factorization Theorem to show that $\boldsymbol{T}(\boldsymbol{X})$ is a k-dimensional sufficient statistic for $\boldsymbol{\eta}$.

b) Use the Lehmann Scheffé LSM theorem to show that $\boldsymbol{T}(\boldsymbol{X})$ is a minimal sufficient statistic for $\boldsymbol{\eta}$.
(Hint: in a regular exponential family, if $\sum_{i=1}^{k} a_i \eta_i = c$ for all $\boldsymbol{\eta}$ in the natural parameter space for some fixed constants $a_1, ..., a_k$ and c, then $a_1 = \cdots = a_k = 0$.)

2) Let $Y_1, ..., Y_n$ be iid (two parameter) exponential EXP(θ, λ). Find a two dimensional sufficient statistic for (θ, λ).
Note: the statistic $\boldsymbol{T}$ should not depend on (θ, λ).

3) Let $Y_1, ..., Y_n$ be iid gamma(ν, λ). Find a two dimensional sufficient statistic for (ν, λ).
Note: the statistic $\boldsymbol{T}$ should not depend on (ν, λ).

4) Let $Y_1, ..., Y_n$ be iid normal $N(\mu, 1)$. Find a minimal sufficient statistic for μ
a) using exponential family theory.
b) Using the Lehmann Scheffé theorem.
Hint for 4b: Beginning students often fail to factor out terms such as $\exp(-0.5 \sum y_i^2)$ that do not depend on μ. To find a minimal sufficient statistic, it is often useful to first use the Factorization theorem to find a sufficient statistic $\boldsymbol{T}$, then use the Lehmann Scheffé theorem to show that $\boldsymbol{T}$ is also a minimal sufficient statistic.

5) (4.5): If $X_1, ..., X_n$ are iid with $f(x|\theta) = \exp[-(x - \theta)]$ for $x > \theta$, then the joint pdf can be written as

$$f(\boldsymbol{x}|\theta) = e^{n\theta} \exp(-\sum x_i) I[\theta < x_{(1)}].$$

By the Factorization Theorem, $\boldsymbol{T}(\boldsymbol{X}) = (\sum X_i, X_{(1)})$ is a sufficient statistic. Show that $R(\theta) = f(\boldsymbol{x}|\theta)/f(\boldsymbol{y}|\theta)$ can be constant even though $\boldsymbol{T}(\boldsymbol{x}) \neq \boldsymbol{T}(\boldsymbol{y})$. Hence the Lehmann-Scheffé Theorem does not imply that $\boldsymbol{T}(\boldsymbol{X})$ is a minimal sufficient statistic.

6) Suppose $X_1, ..., X_n$ are iid with $f(x|\theta) = \exp[-(x - \theta)]$ for $x > \theta$ where θ is real. Find a minimal sufficient statistic for θ.