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Math 580 HW 7 Spring 2022.  Due Wednesday, March 16.
Quiz 6 on sufficient and minimal sufficient statistics.
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1) Let $X_1, ..., X_n$ be iid uniform U$(\theta, \theta + 1)$.

a) Show that $\boldsymbol{T}(\boldsymbol{X}) = (X_{(1)}, X_{(n)})$ is a minimal sufficient statistic for θ.

b) Show that $\boldsymbol{T}(\boldsymbol{X}) = (X_{(1)}, X_{(n)})$ is not complete (not a complete sufficient statistic).
Hint: find $E(X_{(1)})$ and $E(X_{(n)})$ and then show that $E_\theta(aX_{(1)} + bX_{(n)} + c) = 0$ for all θ for some constants a, b, and c, but $P_\theta(aX_{(1)} + bX_{(n)} + c = 0) = 0 < 1$. These constants can depend on n since n is known. See a similar example done in class. You are showing that there is a function $g(X_{(1)}, X_{(n)})$ such that $E_\theta\ g(X_{(1)}, X_{(n)}) = 0$ for all θ, but $P_\theta(g(T_1, T_2) = 0) = 0 < 1$.)

2) Let $Y_1, ..., Y_n$ be iid with pdf or pmf given below, and find a complete sufficient statistic for θ.
a) Let

$$f(y|\theta) = \frac{\theta}{(1+y)^{1+\theta}}$$

where $y > 0$ and $\theta > 0$.
b) Let

$$f(y|\theta) = e^{-(y-\theta)} \exp(-e^{-(y-\theta)})$$

where y and θ are real.
c) Let

$$f(y|\theta) = \binom{2}{y} \theta^y (1-\theta)^{2-y}$$

where $y = 0, 1, 2$ and $0 < \theta < 1$.
Hint: show that these families are kP-REFs, that is, use Corollary 4.6.

3) (4.4) Let $X_1, ..., X_n$ be iid $N(\mu, \gamma_o^2 \mu^2)$ random variables where $\gamma_o^2 > 0$ is **known** and $\mu > 0$.

a) Find a sufficient statistic for μ.

b) Show that $(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2)$ is a minimal sufficient statistic.

c) Find $E_\mu[\sum_{i=1}^n X_i^2]$.

d) Find $E_\mu[(\sum_{i=1}^n X_i)^2]$.

e) Find

$$E_\mu[\frac{n + \gamma_o^2}{1 + \gamma_o^2} \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2].$$

(Hint: use c) and d).)

f) Is the minimal sufficient statistic given in b) complete? Explain.