Math 580 HW 8 Spring 2022. Due Wednesday, March 30.
Quiz 7 on sufficient, minimal sufficient and complete sufficient statistics.
Exam 2 Wed., March 23 covers through HW7. 1 page 6 problems

1) Let $Y_1, ..., Y_n$ be iid from a location family with pdf $f_Y(y) = f_X(y - \theta)$. Show that $M - \overline{Y}$ is an ancillary statistic where M is the sample median. (Hint: let $X_1, ..., X_n$ iid from $f_X(x)$ where $Y_i = X_i + \theta$. Show that $M - \overline{Y}$ can be written as a function of $X_1, ..., X_n$. Use the fact that $M = MED_n(Y_i) = MED_n(X_i) + \theta$. See Example 4.18.)

2) (5.2) Let (X, Y) have the bivariate density

$$f(x, y) = \frac{1}{2\pi} \exp(\frac{-1}{2}[(x - \rho \cos\theta)^2 + (y - \rho \sin\theta)^2]).$$

Suppose that there are n independent pairs of observations (X_i, Y_i) from the above density and that ρ is known. Assume that $0 \leq \theta \leq 2\pi$. Find a candidate for the maximum likelihood estimator $\hat{\theta}$ by differentiating the log likelihood $\log(L(\theta))$. (Do not show that the candidate is the MLE, it is difficult to tell whether the candidate, 0 or 2π is the MLE without the actual data.)

3) (5.3) Suppose a single observation $X = x$ is observed where X is a random variable with pmf given by the table below. Assume $0 \leq \theta \leq 1$, and find the MLE $\hat{\theta}_{MLE}(x)$. (Hint: drawing $L(\theta) = L(\theta|x)$ for each of the four values of x may help.)

x	1	2	3	4
$f(x\|\theta)$	1/4	1/4	$\frac{1+\theta}{4}$	$\frac{1-\theta}{4}$

4) (5.5) Suppose that $X_1, ..., X_n$ are iid $U(0, \theta)$. Use the factorization theorem to write $f(\boldsymbol{x}|\theta) = g(T(\boldsymbol{x})|\theta) I[x_{(1)} \geq 0]$ where $T(\boldsymbol{x})$ is a one dimensional sufficient statistic. Then plot the likelihood function $L(\theta) = g(T(\boldsymbol{x})|\theta)$ and find the MLE of θ.

5) (almost 5.26) Let $Y_1, ..., Y_n$ be iid with pdf

$$f(y|\theta) = \theta y^{-2}$$

where $0 < \theta \leq y < \infty$.

a) Find a low dimensional sufficient statistic for θ.

b) Find the MLE of θ. (Hint: plot $L(\theta)$.)

6) Let $Y_1, ..., Y_n$ be iid with pdf

$$f(y|\theta) = \frac{1}{2} e^{-|y-\theta|}$$

where y and θ are real. Find the MLE of θ using the fact that the median M minimizes $\sum |Y_i - \theta|$. You need not prove this fact.