Math 580 Quiz 1 Spring ~~2008~~ 2014 makeup 2017 Name____________________

1) Let $Y \sim \text{beta}(\delta, \nu)$. Using the kernel method, find $E(Y^k)$ where $k > -\delta$.

$$= \int_0^1 y^k f(y)\,dy = \int_0^1 y^k \frac{\Gamma(\delta+\nu)}{\Gamma(\delta)\Gamma(\nu)} y^{\delta-1}(1-y)^{\nu-1}\,dy$$

$$= \frac{\Gamma(\delta+\nu)}{\Gamma(\delta)\Gamma(\nu)} \frac{\Gamma(\delta+k)\Gamma(\nu)}{\Gamma(\delta+k+\nu)} \underbrace{\int_0^1 \frac{\Gamma(\delta+k+\nu)}{\Gamma(\delta+k)\Gamma(\nu)} y^{\delta+k-1}(1-y)^{\nu-1}\,dy}_{1 = \int \text{beta}(\delta+k,\nu)\text{ pdf}}$$

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$$= \boxed{\frac{\Gamma(\delta+\nu)\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(\delta+k+\nu)}}$$

2) Suppose Y has mgf

$$m(t) = \frac{\log(1-\theta e^t)}{\log(1-\theta)}$$

for $t < -\log(\theta)$. Find $E(Y)$.

$$m'(t) = \frac{1}{\log(1-\theta)} \frac{1}{1-\theta e^t}(-\theta e^t)$$

$$EY = m'(0) = \boxed{\frac{-\theta}{\log(1-\theta)(1-\theta)}} = \frac{\theta}{(\theta-1)\log(1-\theta)}$$

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3) Let $X \sim N(\mu, \sigma^2)$ so that $EX = \mu$ and Var $X = \sigma^2$.

a) Find $E(X^2)$.

$V(X) + (EX)^2 = \boxed{\sigma^2 + \mu^2}$

b) If $k \geq 2$ is an integer, then $E(X^k) = (k-1)\sigma^2 E(X^{k-2}) + \mu E(X^{k-1})$. Use this recursion relationship to find $E(X^3)$.

$= 2\sigma^2 EX + \mu EX^2$

$= 2\sigma^2\mu + \mu(\sigma^2 + \mu^2)$

$= 3\sigma^2\mu + \mu^3$

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