2017

Math 580 Quiz 11 Spring 2012 Name____________________

1) Let $Y_1, ..., Y_n$ be iid Exponential(λ) random variables. $E(Y) = \lambda$ $V(Y) = \lambda^2$

a) Find the limiting distribution of $\sqrt{n}(\overline{Y}_n - \lambda)$. $\xrightarrow{D} \boxed{N(0, \lambda^2)}$

by CLT

b) Find the limiting distribution of $\sqrt{n}\,[\,(\overline{Y}_n)^2 - c\,]$ for appropriate constant c.

$g(\lambda) = \lambda^2$, $g'(\lambda) = 2\lambda$, $[g'(\lambda)]^2 = 4\lambda^2$

$\sqrt{n}\,[(\overline{Y}_n)^2 - \lambda^2] \xrightarrow{D} N(0, \lambda^2 4\lambda^2) = \underline{N(0, 4\lambda^4)}$ by delta method

c) Consider the likelihood ratio test for $H_0 : \lambda = 1$ versus $H_1 : \lambda \neq 1$. Hence $\theta = \lambda$.
i) What is $\hat{\theta}_0$?

$\hat{\theta}_0 = \hat{\lambda}_0 = 1$

ii) What is $\hat{\theta}$? $\hat{\theta} = \overline{Y}_n$

iii) Write down the rejection region and formula for α in terms of $\lambda(\boldsymbol{x})$. (Do not compute $\lambda(\boldsymbol{x})$.)

reject H_0 if $\lambda(\underline{y}) \le c$

where $P_1(\lambda(\underline{Y}) \le c) = \alpha$

similar to 3100222

2) Let $Y_1, ..., Y_n$ be independent identically distributed random variables from a distribution with pdf

$$\frac{2}{y+y^3} \frac{1}{\lambda} \exp[\underbrace{\frac{-1}{\lambda}}_{w(\lambda)} \log(1+y^{-2})]$$

where $y > 0$ and $\lambda > 0$.

$\left(w'(\lambda) = \frac{1}{\lambda^2} > 0 \right)$

a) What is the UMP (uniformly most powerful) level α test for $H_o : \lambda = 2$ vs. $H_1 : \lambda = 4$?

1PREF reject H_0 if $\sum_{i=1}^{n} \log(1+y_i^{-2}) > k$

where $P_2\left(\sum_{i=1}^{n} \log(1+y_i^{-2}) > k\right) = \alpha$

or NP $\frac{f(\underline{y}|4)}{f(\underline{y}|2)} = \frac{\frac{1}{4^n} \exp[-\frac{1}{4} \sum \log(1+y_i^{-2})]}{\frac{1}{2^n} \exp[-\frac{1}{2} \sum \log(1+y_i^{-2})]} > c$

iff $\exp[(-\frac{1}{4}+\frac{1}{2}) \sum \log(1+y_i^{-2})] > c'$

iff $\sum \log(1+y_i^{-2}) > k$

so

b) If possible, find the UMP level α test for $H_o : \lambda = 2$ vs. $H_1 : \lambda > 2$.

same as in a)

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