makeup Q2 2014

Math 580 Quiz 2 Spring 2004 Name____________________

2017

special case of beta binomial

1) Suppose that the conditional distribution of $Y|P = p$ is the binomial(n, p) distribution and that the random variable P has a beta$(\alpha = 4, \beta = 6)$ distribution.

20 a) Find E(Y). $= E(\underbrace{E\,Y|P}_{\text{bin}(n,P)}) = E(nP)$

2.73

$= n\,E\,P = n\frac{\alpha}{\alpha+\beta} = n\frac{4}{10} = \boxed{0.4\,n}$

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20 b) Find Var(Y). $= E(Var(Y|P)) + Var(E(Y|P))$

ex on notes 9

$= E\left[nP(1-P)\right] + Var(nP)$

$= n\,E(P) - n\underbrace{E(P^2)}_{Var(P)+(E(P))^2} + n^2\,Var(P)$

$= n\frac{\alpha}{\alpha+\beta} - n\left[\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} + \left(\frac{\alpha}{\alpha+\beta}\right)^2\right] + n^2\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$= n\,0.4 - n\left[0.021818 + .16\right] + n^2(.021818)$

$= \underbrace{.021818}_{\frac{12}{55(10)}}\,n^2 + \underbrace{0.21818}_{\frac{12}{55}}\,n$

$= \frac{6}{275}$

M 2) Suppose that X has pdf

$$f(x) = \frac{v(x)e^{\theta x}}{\lambda(\theta)}$$

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for $x \in \mathcal{X}$ and for $-\infty < \theta < \infty$ where $\lambda(\theta)$ is some positive function of θ and $v(x)$ is some nonnegative function of x. Find the moment generating function of X using the kernel method. Your final answer should be written in terms of λ, θ and t. 1.26

$$M(t) = \int_{\mathcal{X}} e^{tx} \frac{v(x)\, e^{\theta x}}{\lambda(\theta)}\, dx = \int_{\mathcal{X}} \frac{v(x)\, e^{(\theta+t)x}}{\lambda(\theta)}\, dx$$

$$= \frac{\lambda(\theta+t)}{\lambda(\theta)} \underbrace{\int_{\mathcal{X}} \frac{v(x)\, e^{(\theta+t)x}}{\lambda(\theta+t)}\, dx}_{1} = \boxed{\frac{\lambda(\theta+t)}{\lambda(\theta)}}$$

Q

1.26

0 3) Suppose that $X \sim$ normal $N(\mu = 1, \sigma^2 = 9)$. Let $W = 5X + 25$. Find EW and Var W.

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$$EW = 5EX + 25 = \boxed{30} \quad 15$$

$$\text{Var}\, W = 25\, \text{Var}\, X = 25(9) = \boxed{225} \quad 15$$

$V(W) = \underbrace{EW^2}_{} - (EW)^2 = 225 + (30)^2 - (30)^2$

$V(W) + (EW)^2$

or $EW^2 = E(5X+25)^2$ 2