2017

Math 580 Quiz 3 Spring 2012 Name___________________

	4	1	1	4
x	−2	−1	1	2
f(x) = P(X = x)	3/20	7/20	4/20	6/20

1) Let the discrete random variable X have a probability mass function given by the table above. a) Find the pmf of $Y = X^2$.

$P(Y=y) =$

y	1	4
$f(y)$	$\frac{11}{20} = .55$	$\frac{9}{20} = .45$

b) Find the moment generating function mgf of Y.

$E e^{tY} = \sum e^{ty} f(y) = \boxed{e^{t1}\frac{11}{20} + e^{t4}\frac{9}{20} = e^{t}\,.55 + e^{t4}\,.45}$

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2) Suppose that X is a random variable with pdf

$$f(x) = \frac{4}{\sqrt{\pi}}\,\gamma^{3/2}\,x^2\,e^{-\gamma x^2}$$

where $x > 0$ and $\gamma > 0$. Find the pdf of $Y = X^2$. Do not forget the support $\mathcal{Y}$.

$Y = X^2 = t(X)$, $t(0) = 0$, $t(\infty) = \infty$ so $\mathcal{Y} = (0, \infty)$

$x = \sqrt{y} = t^{-1}(y) = y^{\frac{1}{2}}$, $\left|\frac{dt^{-1}(y)}{dy}\right| = \left|\frac{1}{2}y^{-\frac{1}{2}}\right| = \frac{1}{2\sqrt{y}}$

$f_Y(y) = f_X(t^{-1}(y)) \left|\frac{dt^{-1}(y)}{dy}\right|$

$= \frac{4}{\sqrt{\pi}}\gamma^{3/2}(\sqrt{y})^2 e^{-\gamma(\sqrt{y})^2}\frac{1}{2\sqrt{y}}$

$$\boxed{= \frac{2}{\sqrt{\pi}}\gamma^{\frac{3}{2}} y^{\frac{1}{2}} e^{-\gamma y}, \quad y > 0 \qquad = \frac{2}{\sqrt{\pi}}\gamma^{3/2}\sqrt{y}\, e^{-\gamma y}, \quad y > 0}$$

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$\left(\Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\Gamma(\tfrac{1}{2}) = \tfrac{1}{2}\sqrt{\pi} \text{ so } \tfrac{1}{\Gamma(3/2)} = \tfrac{2}{\sqrt{\pi}}\right) \quad Y \sim G(\nu = \tfrac{3}{2}, \lambda = \tfrac{1}{\gamma})$

$$\frac{\partial}{\partial y_1} y_2 y_1^{-1} = -\frac{y_2}{y_1^2}$$

3) Suppose that X_1 and X_2 have joint pdf

$$f_{X_1,X_2}(x_1,x_2) = 2(1-x_1)$$

for $0 < x_1 < 1$ and $0 < x_2 < 1$. Consider the transformation $Y_1 = X_1$ and $Y_2 = X_1X_2$.

a) Are X_1 and X_2 independent? Explain briefly.

yes $f(x_1,x_2) = [2(1-x_1)]\,[1] = h_1(x_1)\,h_2(x_2)\;(= f_{x_1}(x_1)\,f_{x_2}(x_2))$

on cross product support 13

b) Find the Jacobian J for the transformation. or -1

$x_1 = y_1 = t_1^{-1}(y_1,y_2)$, $x_2 = \frac{y_2}{y_1} = t_2^{-1}(y_1,y_2)$

$\frac{\partial t_1^{-1}}{\partial y_1} = 1$ $\quad \frac{\partial t_1^{-1}}{\partial y_2} = 0$

$\frac{\partial t_2^{-1}}{\partial y_1} = -\frac{y_2}{y_1^2}$ $\quad \frac{\partial t_2^{-1}}{\partial y_2} = \frac{1}{y_1}$ $\quad$ so $J = \begin{vmatrix} 1 & 0 \\ -\frac{y_2}{y_1^2} & \frac{1}{y_1} \end{vmatrix} = \boxed{\frac{1}{y_1}} = |J|$

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c) Find the joint pdf $f(y_1,y_2)$ of Y_1 and Y_2. Include the support.

$f_{X_1,X_2}(x_1,x_2) = 2(1-x_1)\, I(0<x_1<1)\, I(0<x_2<1)$

$f_{Y_1,Y_2}(y_1,y_2) = f_{X_1,X_2}(t_1^{-1}(y),\, t_2^{-1}(y))\,|J| = f_{X_1,X_2}\left(y_1, \frac{y_2}{y_1}\right)\frac{1}{y_1} =$

$\frac{2}{y_1}(1-y_1)\, I(0<y_1<1)\, I\left(0<\frac{y_2}{y_1}<1\right) = 2\,\frac{1-y_1}{y_1}\, I(0<y_1<1)\, I(0<y_2<y_1)$

$$= 2\,\frac{1-y_1}{y_1}\, I(0<y_2<y_1<1)$$ 12

no support, −4

d) Are Y_1 and Y_2 independent? Explain briefly.

no the support is not a cross product

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