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Math 580 Quiz 4 Spring 2008 Name____________________

1) Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$.

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Let $\sigma_{12} = Cov(Y, X)$ and suppose Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2\left(\begin{pmatrix} 16 \\ 25 \end{pmatrix}, \begin{pmatrix} 100 & \sigma_{12} \\ \sigma_{12} & 144 \end{pmatrix} \right).$$

a) when are Y and X ind? soln if $\boxed{\sigma_{12}=0}$

a) If $\sigma_{12} = 0$, find the conditional distribution of $Y|X$. Explain your reasoning.

$\boxed{Y|X \sim N(16, 100) \quad \text{by ind}}$

or since $E(Y|X) = 16 + 0$

$V(Y|X) = 100 - 0$

b) If $\sigma_{12} = 9$, find $E(Y|X)$. $= 16 + \frac{9}{144}(X - 25)$

$= 16 - \frac{9(25)}{144} + \frac{9}{144}X = \boxed{14.4375 + 0.0625\,X}$

c) If $\sigma_{12} = 9$, find Var$(Y|X)$. $= 100 - \frac{9(1)9}{144} =$

$100 - \frac{81}{144} = \boxed{99.4375}$

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4B

Poor problem for correlation

2) Let $X_1, ..., X_n$ be mutually independent random variables such that X_i is normal $N(\mu_i, \sigma_i^2)$. Let $W = \sum_{i=1}^n X_i$. Use moment generating functions to find the distribution of W.

$$M_W(t) = \prod_{i=1}^n m_{X_i}(t) = \prod_{i=1}^n \exp\left(\mu_i t + \frac{\sigma_i^2 t^2}{2}\right)$$

$$= \exp\left[\left(\sum_{i=1}^n \mu_i\right) t + \left(\sum_{i=1}^n \sigma_i^2\right) \frac{t^2}{2}\right]$$

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So $W \sim N\left(\sum_{i=1}^n \mu_i, \sum_{i=1}^n \sigma_i^2\right)$

3) Let $Y_1, ..., Y_n$ be iid $N(a\sigma, \sigma^2)$ random variables where a is a **known** real number and $\sigma > 0$.

13 a) Find $E[\sum_{i=1}^n Y_i^2]$. $= \sum_{i=1}^n E Y_i^2 = n E(Y_1^2) = n\left[V(Y_1) + (E Y_1)^2\right]$

$$= \boxed{n(\sigma^2 + a^2\sigma^2) = n\sigma^2(1 + a^2)}$$

13 b) Find $E[(\sum_{i=1}^n Y_i)^2]$. (Hint: the distribution of $W = \sum Y_i$ can be found using Problem 2).)

$W = \sum Y_i \sim N(na\sigma, n\sigma^2)$

$$= E W^2 = V(W) + (EW)^2$$

$$= \boxed{n\sigma^2 + n^2 a^2 \sigma^2 = n\sigma^2(1 + na^2)}$$

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$$\left(= \sum V Y_i + \left[\sum E Y_i\right]^2 = n\sigma^2 + (na\sigma)^2\right)$$