2017

Math 580 Quiz 5 Spring 2012 Name______________________

1) Let $Y_{(n)} = \max_{1 \le i \le n} Y_i$. If $Y_1, ..., Y_n$ are iid random variables with pdf $f_Y(y) = 2y$ for $0 < y < 1$, find $E(Y_{(n)})$.

$F(y) = \int_0^y 2t\,dt = 2\frac{t^2}{2}\Big|_0^y = y^2, \quad 0<y<1$

give support

$f_{Y_{(n)}}(y) = n[F(y)]^{n-1} f(y) = n(y^2)^{n-1} 2y = 2n\, y^{2n-2+1}$

$= 2n\, y^{2n-1}, \quad 0<y<1$

So $E[Y_{(n)}] = \int_0^1 y\, 2n\, y^{2n-1}\,dy = 2n\int_0^1 y^{2n}\,dy = 2n \frac{y^{2n+1}}{2n+1}\Big|_0^1$

$= \boxed{\dfrac{2n}{2n+1}}$

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$\left(\log(n) + \log(\theta) = \log(n\theta) = \log(\theta + \cdots + \theta) \ne [\log\theta]^n \right.$

$\left. 2.9957 = \log(20) \ne [\log(10)]^2 = 5.3018 \right)$

2) Let $Y_1, ..., Y_n$ be iid with pdf

$$f(y) = \frac{\log(\theta)}{\theta - 1}\theta^y$$

where $0 < y < 1$ and $\theta > 1$. Find a sufficient statistic $T(\mathbf{Y})$ for θ.

$$f(\underline{y}) = \prod_{i=1}^n f(y_i) = \underbrace{\left[\frac{\log(\theta)}{\theta - 1}\right]^n \theta^{\sum y_i}}_{g(T(\underline{y})|\theta)} \underbrace{I[0 < y_{(1)} < y_{(n)} < 1]}_{h(\underline{y})}$$

or g

So $T(\underline{Y}) = \sum_{i=1}^n Y_i$ is sufficient by factorization.

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usually prefer $\lambda(y) > 0$ to $\lambda(y) < 0$

3) Let $Y_1, ..., Y_n$ be iid gamma $(\frac{1}{\lambda^2}, \lambda)$ random variables with pdf

$$f(y) = \frac{1}{y} \frac{y^{\frac{1}{\lambda^2}} e^{-y/\lambda}}{\lambda^{\frac{1}{\lambda^2}} \Gamma(\frac{1}{\lambda^2})}$$

where λ and y are positive.

a) Show that the family $\{f(y|\lambda) : \lambda > 0\}$ is a two parameter exponential family.

$$f(y) = \underbrace{\frac{1}{y} I(y>0)}_{h(y)} \underbrace{\frac{1}{\lambda^{1/\lambda^2} \Gamma(\frac{1}{\lambda^2})}}_{c(\lambda)} \exp\left[\underbrace{\frac{-1}{\lambda}}_{\eta_1} y + \underbrace{\frac{1}{\lambda^2}}_{\eta_2} \log(y)\right]$$

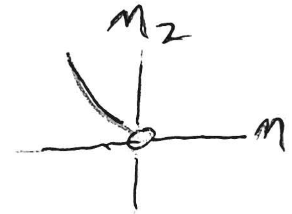
or $-y$

Or $= \underbrace{I(y>0)}_{h(y)} \underbrace{\frac{1}{\lambda^{\frac{1}{\lambda^2}} \Gamma(\frac{1}{\lambda^2})}}_{c(\lambda)} \exp\left[\underbrace{\frac{-1}{\lambda}}_{\eta_1} y + \underbrace{\left(\frac{1}{\lambda^2} - 1\right)}_{\eta_2} \log(y)\right]$

b) Show that the natural parameter space Ω is a parabola. You may assume that $\eta_i = w_i(\lambda)$. Is this family a regular exponential family? Explain briefly.

$\eta_1 = \frac{-1}{\lambda} \in (-\infty, 0)$ $\quad \eta_2 = \frac{1}{\lambda^2} \in (0, \infty)$

So $\eta_2 = \eta_1^2$ and Ω is a parabola



No the family is not regular since Ω is not a 2 dimensional open set.

Or $\eta_1 = -\frac{1}{\lambda} \in (-\infty, 0)$, $\eta_2 = \frac{1}{\lambda^2} - 1 = \eta_1^2 - 1 \in (-1, \infty)$

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or $\eta_1 = \frac{1}{\lambda^2} - 1$ $\quad \eta_2 = \frac{-1}{\lambda}$, $\eta_1 = \eta_2^2 - 1$

or $\eta_1 = \frac{1}{\lambda^2}$ $\quad \eta_2 = \frac{-1}{\lambda}$ $\quad \eta_1 = \eta_2^2$