Math 580 Quiz 6 Spring 2017 Name____________________

1) Suppose that $Y_1, ..., Y_n$ are iid gamma(ν, λ) random variables. Find a minimal sufficient statistic for (ν, λ). Hint: write as a 2 parameter REF.

$$f(y) = \frac{y^{\nu-1}}{\lambda^\nu \Gamma(\nu)} e^{-y/\lambda}, \quad y > 0$$

$$= \underbrace{\frac{1}{\lambda^\nu \Gamma(\nu)}}_{c(\nu,\lambda)} \exp\left[\underbrace{(\nu-1)}_{w_1(\nu,\lambda)} \underbrace{\log(y)}_{t_1(y)} \underbrace{-\frac{1}{\lambda}}_{w_2(\nu,\lambda)} \underbrace{y}_{t_2(y)}\right] \underbrace{I(y>0)}_{h(y)}$$

2PREF so $\underline{T} = \left(\sum \log(Y_i), \sum Y_i\right)$ is min suff

always sums for 2PREF

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2) Let Y be a uniform $U(-\theta, \theta)$ random variable where $\theta > 0$ (and the sample size $n = 1$). Show that $T(Y) = |Y| = \max(-Y, Y)$ is a minimal sufficient statistic.

$$f(y|\theta) = \frac{1}{2\theta} I(-\theta \le y \le \theta) \left(= \frac{1}{2\theta} I(-y \le \theta)\, I(y \le \theta)\right)$$

$$= \frac{1}{2\theta} I\left(\max(-y, y) \le \theta\right) = \frac{1}{2\theta} I(|y| \le \theta).$$

$$\text{so } R_{x,y}(\theta) = \frac{I(|x| \le \theta)}{I(|y| \le \theta)} = c \quad \forall \theta$$

iff $|x| = |y|$ so

$T(Y) = |Y|$ is min suff by LSM

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$f(\underline{y}) = \theta^n \sigma^{n\theta} \exp[-(\theta+1)\sum \log(y_i)] I(y_{(1)} > \sigma) \underbrace{1}_{h(\underline{y})}$

3) Let $Y_1, ..., Y_n$ be iid with pdf $g(T(\underline{Y}) | \theta, \sigma)$

$$f(y) = \theta \sigma^\theta \frac{1}{y^{\theta+1}}$$

so $\left(\sum_{i=1}^n \log Y_i, Y_{(1)}\right)$ is suff by factorization

or

for $y > \sigma$, $\sigma > 0$ and $\theta > 0$. Find a 1 or 2 dimensional sufficient statistic for (θ, σ). Hint: $a = \exp(\log(a))$ where $a = 1/y^{\theta+1} > 0$.

$$f(\underline{y}) = \prod f(y_i) = \underbrace{\theta^n \sigma^{n\theta} \left(\frac{1}{\prod y_i}\right)^{\theta+1} I(y_{(1)} > \sigma)}_{g(T(\underline{Y}) | \theta, \sigma)} \underbrace{1}_{h(\underline{y})}$$

so $\left(\prod_{i=1}^n Y_i, Y_{(1)}\right)$ is suff by Factorization

$$\left(= \underbrace{\prod_{i=1}^n \frac{1}{y_i} I(y_i \ge 0)}_{h(\underline{y})} \overbrace{\theta^n \sigma^{n\theta} \exp[-\theta \sum \log y_i)] I(y_{(1)} > \sigma)}^{g(T(\underline{Y}) | \theta, \sigma) \text{ hard way}} \right)$$

tricky but makes $h(\underline{y}) \ge 0$, $\underline{y} \in \mathbb{R}^n$

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4) Let $Y_1, ..., Y_n$ be iid with pdf $f(y) = \theta y^{\theta-1}$ where $\theta > 0$ and $0 < y < 1$. Find a minimal sufficient statistic for θ using the the Lehmann Scheffé LSM theorem.

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$f(\underline{y}) = \theta \exp[(\theta-1) \log(y_i)] I(0 < y < 1)$

$$R_{\underline{x}, \underline{y}}(\theta) = \frac{f(\underline{x})}{f(\underline{y})} = \frac{\exp[(\theta-1) \sum \log(x_i)]}{\exp[(\theta-1) \sum \log(y_i)]} \underbrace{\frac{I(0 < x_{(1)} < x_{(n)} < 1)}{I(0 < y_{(1)} < y_{(n)} \le 1)}}_{\text{constant wrt } \theta} = c \; \forall \theta$$

iff $\exp((\theta-1)[\sum \log(x_i) - \sum \log(y_i)]) = d \; \forall \theta$

iff $\sum \log(x_i) = \sum \log(y_i)$ so $\boxed{\sum \log(Y_i)}$ is min suff by LSM

$$\text{or } \frac{f(\underline{x})}{f(\underline{y})} = \underbrace{\frac{I[0 < x_{(1)} < x_{(n)} < 1]}{I[0 < y_{(1)} < y_{(n)} < 1]}}_{\text{constant wrt } \theta} \frac{(\prod x_i)^{\theta-1}}{(\prod y_i)^{\theta-1}} = c \; \forall \theta$$

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iff $\left(\frac{\prod x_i}{\prod y_i}\right)^{\theta-1} = d \; \forall \theta$ iff $\frac{\prod x_i}{\prod y_i} \overset{2}{=} 1$ iff $\prod x_i = \prod y_i$

0 is not a possibility

so $\boxed{\prod_{i=1}^n Y_i \text{ is min suff by LSM}}$ (If $\prod x_i = k \prod y_i$, $R = k^{\theta-1} \ne$ constant unless $k = 1$)