or $f(x) = \frac{1}{\sqrt{2\pi\theta}} e^{\frac{-1}{2\theta}(x-\theta)^2} = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{\theta}{2}} e^{x} \exp\left[\frac{-1}{2\theta} x^2\right]$

old quiz 7

Math 580 Quiz 7 Spring 2012 Name____________________

2017

$\Omega = (-\infty, 0)$

1PREF

so $\sum x_i^2$ is complete

so min suff

1) Let $X_1, ..., X_n$ be iid random variables from a normal distribution with mean θ and variance also θ where $\theta > 0$.

10 a) Find a minimal sufficient statistic for θ.

$$\frac{f(\underline{x})}{f(\underline{y})} = \frac{\prod f(x_i)}{\prod f(y_i)} = \frac{\exp\left[\frac{-1}{2\theta} \sum (x_i-\theta)^2\right]}{\exp\left[\frac{-1}{2\theta} \sum (y_i-\theta)^2\right]} = \exp\left[\frac{-1}{2\theta}\left(\sum (x_i-\theta)^2 - \sum (y_i-\theta)^2\right)\right]$$

$$= \exp\left[\frac{-1}{2\theta}\left(\sum x_i^2 - 2\theta \sum x_i + n\theta^2 - \sum y_i^2 + 2\theta \sum y_i - n\theta^2\right)\right]$$

$$= \exp\left[-\frac{1}{2\theta}\left(\sum x_i^2 - \sum y_i^2\right)\right] \exp\left(\sum x_i - \sum y_i\right) = c \quad \forall \theta$$

iff $\sum x_i^2 = \sum y_i^2$ so $T(\underline{x}) = \sum x_i^2$ is min suff by LSM.

30 b) Show $(\overline{X}, S^2)$ is not complete for θ.

$$E_\theta(\bar{X} - S^2) = \theta - \theta = 0 \quad \forall \theta$$

but $P_\theta(\bar{X} - S^2 = 0) = 0 < 1$ so $(\bar{X}, S^2)$ is not complete

or $(\bar{X}, S^2)$ is not a function of the min suff statistic $\sum x_i^2$, so $(\bar{X}, S^2)$ is not min suff and thus not complete.

40

$f(\underline{x},\underline{y}) = \prod_i^n \frac{x_i^{\nu-1} e^{-x_i/\lambda}}{\lambda^\nu \Gamma(\nu)} \prod_{j=1}^m \frac{y_j^{\nu-1} e^{-y_j/(2\lambda)}}{(2\lambda)^\nu \Gamma(\nu)} = \frac{\exp[(\nu-1)(\sum_i^n \log x_i + \sum_j^m \log y_j) - \frac{1}{\lambda}(\sum_i x_i + \sum_j \frac{y_j}{2})]}{\lambda^{n\nu}(\Gamma(\nu))^n (2\lambda)^{m\nu}(\Gamma(\nu))^m}$

LSM would use $\frac{f(\underline{x},\underline{y})}{f(\underline{w},\underline{z})}$

$x_i > 0$
$y_i > 0$

2) Let $X_1, ..., X_n$ be iid gamma(ν, λ) random variables and $Y_1, ..., Y_m$ iid gamma$(\nu, 2\lambda)$ random variables. Assume that the Y_i's and X_j's are independent. Show that the statistic $(\overline{X}, \overline{Y})$ is not a complete sufficient statistic.

$E_{\nu,\lambda}(2\overline{X} - \overline{Y}) = 2\nu\lambda - 2\nu\lambda = 0 \quad \forall \nu,\lambda$

$E_{\nu,\lambda}(\overline{X} - \frac{1}{2}\overline{Y}) = \nu\lambda - \frac{1}{2} 2\nu\lambda = 0 \quad \forall \nu,\lambda$

but $P_\theta(\overline{X} - \frac{1}{2}\overline{Y} = 0) = 0 < 1$ so $(\overline{X}, \overline{Y})$ is not complete

or $\left(\sum_i^n \log x_i + \sum_j^m \log y_j, \sum_{i=1}^n x_i + \sum_{j=1}^m \frac{y_j}{2}\right) = T_2$ is complete $(\overline{X}, \overline{Y})$ is not a function of T_2 so $(\overline{X}, \overline{Y})$ is not min suff and thus not complete

or by factorization thm $f(\underline{x}) \neq h(\underline{x})\, g(\underline{T}(\underline{x}), \theta)$ where $\underline{T}(\underline{x}) = (\overline{x}, \overline{y})$ or $(\overline{x}, \overline{y})$

30

3) Let $Y_1, ..., Y_n$ be iid random variables from a distribution with pdf

$$f(y) = \frac{\theta}{2(1 + |y|)^{\theta+1}}$$

where $\theta > 0$ and y is real. Then $W = \underbrace{\log(1 + |Y|)}_{w_i}$ has pdf $f(w) = \theta e^{-w\theta}$ for $w > 0$. Find a complete sufficient statistic for θ.

$$f(y) = \underbrace{\frac{\theta}{2}}_{c(\theta)} \exp[\underbrace{-(\theta+1)}_{w(\theta)} \underbrace{\log(1+|y|)}_{t(y)}] \underbrace{1}_{h(y)}$$

$\Omega = (-\infty, -1)$ so 1P REF

$\therefore \sum_{i=1}^n \log(1 + |Y_i|)$ is complete by REF theory or cor 4.6.

$= \sum_{i=1}^n w_i$

30

$f(y) = \frac{\theta}{2} \frac{1}{1+|y|} \exp[-\theta \log(1+|y|)]$