Math 580 Quiz 8 Spring ~~2012~~ 2017 Name____________________

1) Let $Y_1, ..., Y_n$ be independent identically distributed (iid) random variables from a distribution with probability density function (pdf)

$$f(y) = \phi \, y^{-(\phi+1)} \frac{1}{1+y^{-\phi}} \frac{1}{\lambda} \exp[\frac{-1}{\lambda} \log(1+y^{-\phi})]$$

where $y > 0, \phi > 0$ is **known** and $\lambda > 0$.

Sept 2011 Qual

a) Find the maximum likelihood estimator (MLE) of λ.

MLE is maximized on support. write MLE on support, so omit $I(y_{(1)}>0)$ free of λ

$$L(\lambda) = \prod f(y_i) = \phi^n \prod \frac{y_i^{-(\phi+1)}}{1+y_i^{-\phi}} \frac{1}{\lambda^n} \exp\left[\frac{-1}{\lambda} \sum \log(1+y_i^{-\phi})\right]$$

$$\log L(\lambda) = \log\left(\phi^n \prod \frac{y_i^{-(\phi+1)}}{1+y_i^{-\phi}}\right) - n\log\lambda - \frac{1}{\lambda} \sum \log(1+y_i^{-\phi})$$

$$\frac{d \log L(\lambda)}{d\lambda} = \frac{-n}{\lambda} + \frac{\sum \log(1+y_i^{-\phi})}{\lambda^2} \overset{set}{=} 0$$

or $\sum \log(1+y_i^{-\phi}) = n\lambda$ or $\boxed{\hat{\lambda} = \frac{\sum \log(1+Y_i^{-\phi})}{n}}$, unique

or -3

$$\frac{d^2 \log L(\lambda)}{d\lambda^2} = \frac{n}{\lambda^2} - \frac{2\sum \log(1+y_i^{-\phi})}{\lambda^3} \Big|_{\hat{\lambda}} = \frac{n}{\hat{\lambda}^2} - \frac{2n\hat{\lambda}}{\hat{\lambda}^3} = \frac{-n}{\hat{\lambda}^2} < 0$$

or -4

so $\hat{\lambda}$ is the MLE

b) Find the MLE of λ^2.

$\boxed{\hat{\lambda}^2 \text{ by invariance}}$

$$\hat{\lambda}^2 = \left(\frac{\sum \log(1+Y_i^{-\phi})}{n}\right)^2$$

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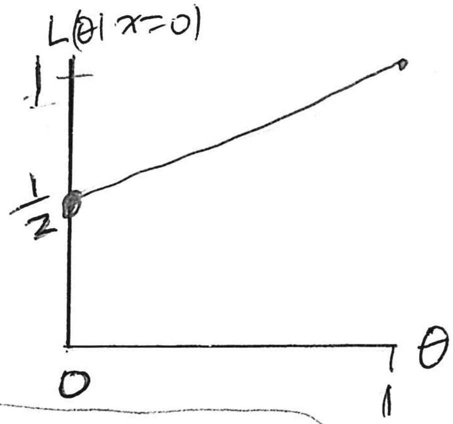
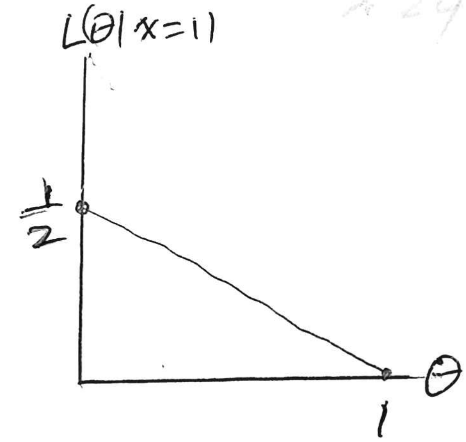
2) Suppose a single observation $X = x$ is observed where X is a random variable with pmf given by the table below. Assume $0 \le \theta \le 1$. and find the MLE $\hat{\theta}_{MLE}(x)$. (Hint: drawing $L(\theta) = L(\theta|x)$ for each of the values of x may help.)

x	0	1
$f(x\|\theta)$	$\frac{1+\theta}{2}$	$\frac{1-\theta}{2}$





$$\boxed{\hat{\theta}(x) = \begin{cases} 1 & x=0 \\ 0 & x=1 \end{cases}}$$

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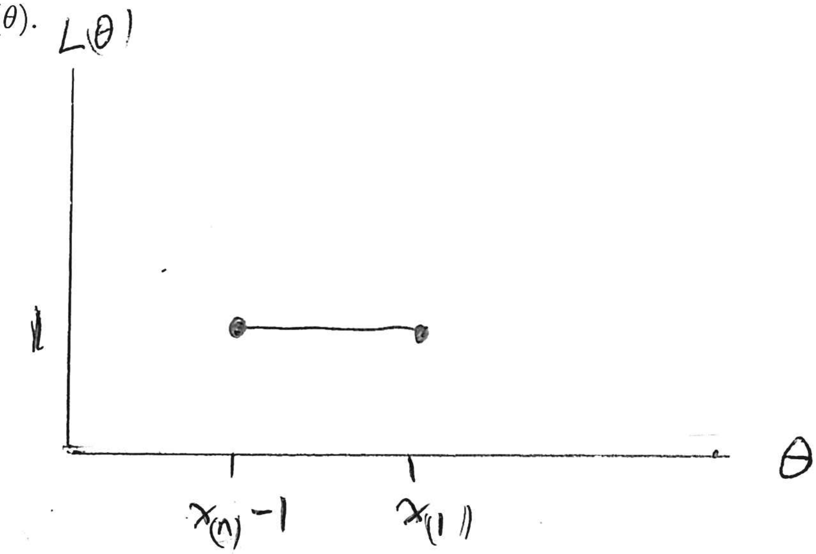
3) Suppose that $X_1, ..., X_n$ are iid $U(\theta, \theta+1)$ so that

$$L(\theta) = 1^n I[x_{(1)} \ge \theta] I[x_{(n)} \le \theta + 1] = I[x_{(n)} - 1 \le \theta \le x_{(1)}].$$

a) Sketch $L(\theta)$.



b) An MLE of θ is $\hat{\theta}_{MLE}(\boldsymbol{x}) = t$ for some fixed $t \in [c, d]$. Find $[c, d]$.

$= [x_{(n)}-1, x_{(1)}]$

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