Math 580 Quiz 9 Spring 2017 Name____________________

1) Let $Y_1, ..., Y_n$ be iid Poisson(θ) random variables.

20 a) Find the method of moments estimator for θ.

$E(Y) = \theta \overset{set}{=} \overline{Y}$ so $\boxed{\hat{\theta} = \overline{Y}}$

20 b) Find the method of moments estimator for $1/\theta$.

$g(\theta) = \frac{1}{\theta}$ so $\boxed{\widehat{\left(\frac{1}{\theta}\right)} = \frac{1}{\overline{Y}}} = g(\overline{Y})$

by mm Theorem

5 c) The Fisher information $I_1(\theta) = 1/\theta$. Find $I_n(1/\theta)$ $= \frac{n\, I_1(\theta)}{[\tau'(\theta)]^2} = \frac{n}{\theta} \frac{1}{\theta^{-4}}$

$\tau(\theta) = \theta^{-1}$, $\tau'(\theta) = -\theta^{-2}$

$= \boxed{n\theta^3}$

10 d) Find the FCRLB for unbiased estimators of $\tau(\theta) = 1/\theta$.

$\frac{\theta^{-4}}{n \frac{1}{\theta}} = \frac{[\tau'(\theta)]^2}{n\, I_1(\theta)} = \boxed{\frac{1}{n\theta^3}} = \frac{1}{I_n(\theta)}$

5 e) The UMVUE for $e^{a\theta}$ is $U_n = \exp[\sum_{i=1}^n Y_i \log(\frac{a+n}{n})]$. Do you expect the UMVUE U_n to achieve the FCRLB for unbiased estimators of $e^{a\theta}$? Explain briefly.

[illegible] $a > -n$ for log

No, $T_n = \sum_{i=1}^n Y_i$ is the complete sufficient statistic

and U_n is a nonlinear function of T_n.

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$U_n = \left(\frac{a+n}{n}\right)^T$ is better then don't need $a > -n$

2) Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y) = \frac{2y}{\theta^2}, \quad 0 < y < \theta.$$

Then $E(Y_i) = 2\theta/3$ and $V(Y_i) = \theta^2/18$. Let $T = c\overline{Y}$ be an estimator of θ where c is a constant. Find the mean square error (MSE) of T as a function of c (and of θ and n).

$E(T) = cE(\overline{Y}) = c\frac{2}{3}\theta$, $V(T) = c^2 V(\overline{Y}) = c^2 \frac{V(Y)}{n} = \frac{c^2\theta^2}{18n}$

$MSE(T) = V(T) + [bias(T)]^2 = \frac{c^2\theta^2}{18n} + (c\frac{2}{3}\theta - \theta)^2$

$= \frac{c^2\theta^2}{18n} + \frac{\theta^2}{9}(2c-3)^2 = \frac{c^2\theta^2}{18n} + \theta^2(\frac{2c}{3} - 1)^2$

$= \frac{c^2\theta^2}{18n} + \frac{\theta^2}{9}(2c-3)^2$

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3) Let $X_1, ..., X_n$ be independent identically distributed random variables with pdf

$$f(x) = \frac{1}{\lambda}\phi x^{\phi-1}\frac{1}{1+x^\phi}\exp\left[-\frac{1}{\lambda}\log(1+x^\phi)\right]$$

where x, ϕ, and λ are all positive. ASSUME ϕ IS KNOWN. Find the uniformly minimum unbiased estimator (UMVUE) of λ.

Hint: You may use the fact that $\log(1+X_i^\phi) \sim$ Gamma $(\nu = 1, \lambda)$.

1 PREF $T = \sum_{i=1}^n \log(1+x_i^\phi) \sim G(n, \lambda)$ is complete

$ET = n\lambda = \sum_{i=1}^n E[\log(1+x_i^\phi)] = \sum_{i=1}^n \lambda = n\lambda$

So $\boxed{\frac{T}{n} = \frac{\sum_{i=1}^n \log(1+x_i^\phi)}{n}}$ is the UMVUE by LSU

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