

Math 581 HW 10 Fall 2021. Due **Tuesday**, Nov. 16.

Exam 2 and 3 reviews may be useful. For quiz 10, the exam reviews and oral exam problems from the course website may be useful. 5 sheets of notes for the quiz.

Final: Wednesday, Dec. 8, 12:30-2:30.

1) (27.4 b) modified slightly): For each $n \in \mathbb{N}$, let W_{nk} be independent with $E(W_{nk}) = 0$, $V(W_{nk}) = \sigma_{nk}^2$, and $s_n^2 = \sum_{k=1}^{r_n} \sigma_{nk}^2$. Suppose $|W_{nk}| \leq M_n$ wp1 and $M_n/s_n \rightarrow 0$. Verify that Lindeberg's condition holds. Show directly: do not use the fact that if Lyapounov's condition holds, then Lindeberg's condition holds.

2) Suppose that X_1, \dots, X_n are iid and $V(X_1) = \sigma^2$. Given that

$$\hat{\sigma}_n^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \xrightarrow{P} \sigma^2,$$

give a very short proof that the sample variance

$$S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \xrightarrow{P} \sigma^2.$$

3) Suppose $\mathbf{X}_1, \dots, \mathbf{X}_n$ are iid $p \times 1$ random vectors from a multivariate t-distribution with parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ with d degrees of freedom. Then $E(\mathbf{X}_i) = \boldsymbol{\mu}$ and $\text{Cov}(\mathbf{X}) = \frac{d}{d-2} \boldsymbol{\Sigma}$ for $d > 2$. Assuming $d > 2$, find the limiting distribution of $\sqrt{n}(\bar{\mathbf{X}} - \mathbf{c})$ for appropriate vector \mathbf{c} .

4) Suppose

$$Z_n = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \xrightarrow{D} N(0, 1)$$

and $s_n^2 \xrightarrow{P} \sigma^2$ where $\sigma > 0$. Prove that

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{s_n} \xrightarrow{D} N(0, 1).$$

5) If $Y_n \xrightarrow{D} Y$, $a_n \xrightarrow{P} a$, and $b_n \xrightarrow{P} b$, then $a_n + b_n Y_n \xrightarrow{D} X$. Find X .