Math 581 HW 11 Fall 2021. Due **Thursday**, Nov. 18.

Exam 3 review may be useful. For quiz 11, the exam reviews and oral exam problems from the course website may be useful. 6 sheets of notes for the quiz.

Exam 3: Tuesday, Nov. 30.

Final: Wednesday, Dec. 8, 12:30-2:30.

1) Let $\boldsymbol{X}_1, ..., \boldsymbol{X}_n$ be iid $k \times 1$ random vectors where $E(\boldsymbol{X}_i) = (\lambda_1, ..., \lambda_k)^T$ and $Cov(\boldsymbol{X}_i) = diag(\lambda_1^2, ..., \lambda_k^2)$, a diagonal $k \times k$ matrix with jth diagonal entry λ_j^2. The nondiagonal entries are 0. Find the limiting distribution of $\sqrt{n}(\overline{\boldsymbol{X}} - \boldsymbol{c})$ for appropriate vector $\boldsymbol{c}$.

2) What theorem can be used to prove both the (usual) contral limit theorem and the Lyapounov CLT?

3) What theorem can be used to prove the existence of $P[A|\mathbb{G}]$ and $E[X|\mathbb{G}]$?

4) Using $E[I_A|\mathbb{G}] = P[A|\mathbb{G}]$ wp1, use $X = I_A$, $Y = I_B$, $X_i = I_{A_i}$, and the result for $E[X|\mathbb{G}]$ to get the corresponding result for $P[A|\mathbb{G}]$.

a) Using $E[\sum_{i=1}^n a_i X_i|\mathbb{G}] = \sum_{i=1}^n a_i E[\sum_{i=1}^n a_i X_i|\mathbb{G}]$, find $E[\sum_{i=1}^n a_i I_{A_i}|\mathbb{G}]$ in terms of $P[A_i|\mathbb{G}]$.

b) If $X \leq Y$ wp1, then $E[X|\mathbb{G}] \leq E[Y|\mathbb{G}]$ wp1. If $A \subseteq B$, then $I_A \leq I_B$. Use these results to show that if $A \subseteq B$, then $P[A|\mathbb{G}] \leq P[B|\mathbb{G}]$ wp1.

c) If $X = a$ wp1, then $E[X|\mathbb{G}] = a$ wp1. Use $1 = I_\Omega$ and b) with $B = \Omega$ to prove $P[A|\mathbb{G}] \leq 1$ wp1.

5) Let a be a constant. Prove $E[aX|\mathbb{G}] = aE[X|\mathbb{G}]$ wp1.