Math 581 HW 7 Fall 2021. Due Thursday, Oct. 14.

Exam 1 and 2 reviews may be useful. For quiz 7, the exam reviews and oral exam problems from the course website may be useful. 5 sheets of notes for the quiz. Exam 2 is Thursday, Oct. 21. The URL (http://parker.ad.siu.edu/Olive/ich8.pdf) has large sample theory at the Math 580 level (lower than that of Math 581), but shows some of the types of problems that I ask.

Final: Wednesday, Dec. 8, 12:30-2:30.

1) Let μ be a measure on $(\Omega, \mathcal{F})$ and let $c > 0$. Prove that $\nu = c\mu$ is measure on $(\Omega, \mathcal{F})$.

Note: If $\mu = \prod_{i=1}^n \mu_i$ is a product measure, then $\nu = c^n \mu = \prod_{i=1}^n c\mu_i = \prod_{i=1}^n \nu_i$ is a product measure by Problem 1). Also, a finite measure $\mu = P/c$ is a scaled probability measure $\nu = P = c\mu$ with $c = 1/\mu(\Omega)$.

2) The random variable X is a point mass at the real number c if $P(X = c) = 1$. Then the pmf $p_X(x) > 0$ only at $x = c$. If h is a (measurable) function, find $E[h(X)]$.

3) Let $X_n \sim U(-n, n)$ have cdf $F_n(x)$. Then $lim_n \, F_n(x) = 0.5$ for all real x. Does $X_n \xrightarrow{D} X$ for some random variable X. Explain briefly.

4) Let X_n be a sequence of random variables such that $P(X_n = 1/n) = 1$. Does X_n converge in law? If yes, prove it by finding X and the cdf of X. If no, prove it.

5) Suppose X_n has cdf

$$F_n(x) = 1 - \left(1 - \frac{x}{\theta n}\right)^n$$

for $x \geq 0$ and $F_n(x) = 0$ for $x < 0$. Show that $X_n \xrightarrow{L} X$ by finding the cdf of X.

6) Suppose that $Y_1, ..., Y_n$ are iid with $E(Y) = (1-\rho)/\rho$ and $\text{VAR}(Y) = (1-\rho)/\rho^2$ where $0 < \rho < 1$. Find the limiting distribution of $\sqrt{n}\, \left(\overline{Y}_n - \dfrac{1-\rho}{\rho}\right)$.

7) Let $X_1, ..., X_n$ be iid with cdf $F(x) = P(X \leq x)$. Let $Y_i = I(X_i \leq x)$ where the indicator equals 1 if $X_i \leq x$ and 0, otherwise.

a) Find $E(Y_i)$.

b) Find $\text{VAR}(Y_i)$.

c) Let $\hat{F}_n(x) = \dfrac{1}{n} \displaystyle\sum_{i=1}^n I(X_i \leq x)$ for some fixed real number x. Find the limiting distribution of $\sqrt{n}\, \left(\hat{F}_n(x) - c_x\right)$ for an appropriate constant c_x.

8) Let $X_n \sim \text{Binomial}(n, p)$ where the positive integer n is large and $0 < p < 1$.

Find the limiting distribution of $\sqrt{n}\, \left(\dfrac{X_n}{n} - p\right)$.