Math 581 HW 9 Fall 2021. Due Thursday, Nov. 5.

Exam 2 and 3 reviews may be useful. For quiz 9, the exam reviews and oral exam problems from the course website may be useful. 5 sheets of notes for the quiz.

Final: Wednesday, Dec. 8, 12:30-2:30.

1) A much better proof for showing convergence in rth mean implies convergence in probability is given in this problem. If $h(t)$ is an increasing function (at least on the range of W), then $P(W \geq c) = P(h(W) \geq h(c))$. Let $\epsilon > 0$. Then $P(|X_n - X| \geq \epsilon) = P(|X_n - X|^r \geq \epsilon^r)$. Now apply the Generalized Chebyshev's Inequality from Exam 1 review 23) to show that if $X_n \xrightarrow{r} X$, then $P(|X_n - X| \geq \epsilon) \to 0$ as $n \to \infty$.

2) For each $n \in \mathbb{N}$, let $X_{n1}, ..., X_{nr}$ be independent RVs on probability space $(\Omega_n, \mathcal{F}_n, P_n)$ with $E(X_{nk}) = \mu_{nk}$, $V(X_{nk}) = \sigma^2_{nk}$, $T_n = \sum_{k=1}^{r_n} X_{nk}$, $E(T_n) = \mu_n = \sum_{k=1}^{r_n} \mu_{nk}$, and $V(T_n) = \sigma_n^2 = \sum_{k=1}^{r_n} \sigma^2_{nk}$.

a) If $v_n > 0$ and $\sigma_n / v_n \to 0$ as $n \to \infty$, use Chebyshev's inequality to prove

$$P_n \left[\left| \frac{T_n - \mu_n}{v_n} \right| \geq \epsilon \right] \to 0$$

$\forall \, \epsilon > 0$ as $n \to \infty$.

b) (6.5 slightly modified): Let $A_1, A_2, ...$ be independent events with $P(A_i) = p_i$ and $\overline{p}_n = \frac{1}{n} \sum_{i=1}^n p_i$. Let $X_{nk} = X_k = I_{A_k}$ and $T_n = \sum_{k=1}^n X_k = \sum_{k=1}^n I_{A_k}$. Let $r_n = n$ and $P_n = P$ for all n. Use a) to prove

$$P[|n^{-1} T_n - \overline{p}_n| \geq \epsilon] \to 0$$

for all $\epsilon > 0$ as $n \to \infty$.

3) (27.4 a) modified slightly): For each $n \in \mathbb{N}$, let W_{nk} be independent with $E(W_{nk}) = 0$, $V(W_{nk}) = \sigma^2_{nk}$, and $s_n^2 = \sum_{k=1}^{r_n} \sigma^2_{nk}$. Suppose $|W_{nk}| \leq M_n$ wp1 and $M_n / s_n \to 0$. Verify that Lyapounov's condition holds.

Hint: $|W_{nk}|^{2+\delta} \leq M_n^\delta W_{nk}^2$ wp1 for $\delta > 0$. Take expectations of both sides.

4) Let $Y_n \sim \chi^2_n$. Find the limiting distribution of $\sqrt{n} \left(\frac{Y_n}{n} - 1 \right)$.

Hint: See exam 2 review 92).

5) Suppose that $X_1, ..., X_n$ are iid and that t is a function such that $E(t(X_1)) = \mu_t$. Is there a constant c such that

$$\frac{\sum_{i=1}^n t(X_i)}{n} \xrightarrow{P} c \ \ ?$$

Explain briefly.

Hint: Similar to Quiz 8 3a).

6) Let $P(X_n = n) = 1$.

a) Show $F_{X_n}(x) \to H(x)$ as $n \to \infty$.

b) Let $M_{X_n}(t)$ be the moment generating function of X_n. Find $lim_n M_{X_n}(t)$ for all t.

Hint: examine $t < 0$, $t = 0$, and $t > 0$.

c) Does X_n converge in distribution?